CHAPTER 5

Rules of Exponents

OUTLINE

Math at Work:

Motorcycle Customization

Let's take another look at how math is used in customizing motorcycles. One of Frank Esposito's coworkers at the motorcycle shop is Tanya Sorello, who is responsible for the frames of the motorcycles. Like Frank, Tanya uses mathematical equations to ensure that she does her work correctly.

For example, Tanya is building a chopper frame and needs to make the supports for the axle. To do this, she has to punch 1-in.-diameter holes in $\frac{3}{8}$-in.-thick mild steel plates that will be welded to the frame. The press punches two holes at a time. To determine how much power is needed to do this job, she uses a formula containing an exponent, $P = \frac{t^2 dN}{3.78}$. After substituting the numbers into the expression, she calculates that the power needed to punch these holes is 0.07 hp.

Tanya is used to working under tight deadlines. She has found ways to manage stress and perform at her best when it matters most. Tanya says she learned these skills when she was in school, during exam time. Mastering the high-pressure situation of sitting down to take a final exam has helped her succeed in critical moments throughout her career.

In this chapter, we will learn about working with exponents and introduce strategies you can use when you take a math test.

POWER

Study Strategies

Taking Math Tests

You've attended class regularly, you've taken good notes, you've done all the assigned homework and problem sets. Yet to succeed in a math course, you still need to do well on the tests. For many students, this makes taking math tests extremely stressful. Keep in mind, though, that all the work you do throughout the term—the note-taking, the homework, all of it—represents test preparation. The strategies below will also help you perform your best on math tests.

P Prepare

- Practice, practice, practice! Repetition will help build your skills. Use a timer as you work on problems to get used to doing math against the clock.
- Complete the test preparation checklist on page 340.
- During sleep, short-term memory turns into long-term memory, so get a good night's sleep before the test.

O Organize

- Warm up for the test just like athletes do before they play a game. No matter how much you studied the night before, do several "warm-up" problems the same day as the test. This way, you will be in the groove of doing math and won't go into the test cold.
- Get to the location of the test at least 10 minutes early.

W Work

- When you receive the test, read all the instructions.
- Look over the entire test, and answer the easiest questions first. This will build your confidence and leave you more time to work on the harder problems.
- Show your work in a neat and organized way. Your instructor may give you partial credit if you show the steps you're going through to arrive at an answer.
- If you start to feel anxiety, don't let it develop into panic. Take deep breaths and trust that your preparation has positioned you to succeed.

E Evaluate

- Leave time at the end of the test to check over your calculations and to make sure you didn't skip any problems.
- If you realize that you will not have time to complete the test, don't give up. Use the time you have to answer as many questions as you can.

R Rethink

- When you get your test back, regardless of your grade, look it over and see where you made errors. Remember, math concepts build on each other—you can't master the new concepts until you are comfortable with the previous ones.
- If you aren't happy with your grade, talk to your instructor. She or he will likely have good advice for how you can do better next time.

Chapter 5 POWER Plan

P Prepare | **O Organize**

What are your goals for Chapter 5?	How can you accomplish each goal?
1 Be prepared before and during class.	• Don't stay out late the night before, and be sure to set your alarm clock! • Bring a pencil, notebook paper, and textbook to class. • Avoid distractions by turning off your cell phone during class. • Pay attention, take good notes, and ask questions. • Complete your homework on time, and ask questions on problems you do not understand.
2 Understand the homework to the point where you could do it without needing any help or hints.	• Read the directions and show all your steps. • Go to the professor's office for help. • Rework homework and quiz problems, and find similar problems for practice.
3 Use the P.O.W.E.R. framework to help you take math tests: *Math Test Preparation Checklist.*	• Read the Study Strategy that explains how to better take math tests. • Where can you begin using these techniques? • Complete the emPOWERme that appears before the Chapter Summary.
4 Write your own goal. ____________________ ____________________	• ____________________________ ____________________________
What are your objectives for Chapter 5?	**How can you accomplish each objective?**

1 Know how to use the product rule, power rules, and the quotient rule for exponents.	• Learn how to use each rule by itself first.
	  • Product rule
	  • Basic power rule
	  • Power rule for a product
	  • Power rule for quotient
	  • Quotient rule for exponents
	• Be able to simplify an expression using any combination of the rules.

2 Understand how to work with negative exponents and zero as an exponent.	• Know that any nonzero number or variable raised to the 0 power is equal to 1. • Learn the definition of a *negative exponent,* and be able to explain it in your own words. • Know how to write an expression containing negative exponents with positive exponents.
3 Use, write, and evaluate numbers in scientific notation.	• Know how to multiply by a positive or negative power of 10 and compare it to using scientific notation. • Understand the definition of scientific notation. • Learn to write any number in scientific notation. • Be able to multiply, divide, add, and subtract numbers in scientific notation.
4 Write your own goal. ____________________ ____________________	• ____________________________ ____________________________

W Work	Read Sections 5.1–5.4 and complete the exercises.
E Evaluate Complete the Chapter Review and Chapter Test. How did you do?	**R Rethink** • Which objective was the hardest for you to master? What steps did you take to overcome difficulties in becoming an expert at that objective? • How has the Study Strategy for this chapter helped you take a more proactive approach to studying for your math tests?

5.1A Basic Rules of Exponents: The Product Rule and Power Rules

P Prepare | **O Organize**

What are your objectives for Section 5.1A?	How can you accomplish each objective?
1 Evaluate Exponential Expressions	• Write the definition of *exponential expression* in your own words, using the words *base* and *exponent*. • Complete the given example on your own. • Complete You Try 1.
2 Use the Product Rule for Exponents	• Learn the property for the **Product Rule** and write an example in your notes. • Complete the given example on your own. • Complete You Try 2.
3 Use the Power Rule $(a^m)^n = a^{mn}$	• Learn the property for the **Basic Power Rule** and write an example in your notes. • Complete the given example on your own. • Complete You Try 3.
4 Use the Power Rule $(ab)^n = a^n b^n$	• Learn the property for the **Power Rule for a Product** and write an example in your notes. • Complete the given example on your own. • Complete You Try 4.
5 Use the Power Rule $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ where $b \neq 0$	• Learn the property for the **Power Rule for a Quotient** and write an example in your notes. • Complete the given example on your own. • Complete You Try 5. • Write the summary for the product and power rules of exponents in your notes, and be able to apply them.

W Work **Read the explanations, follow the examples, take notes, and complete the You Trys.**

1 Evaluate Exponential Expressions

Recall from Chapter 1 that exponential notation is used as a shorthand way to represent a multiplication problem. For example, $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ can be written as 3^5.

Definition

An **exponential expression** of the form a^n, where a is any real number and n is a positive integer, is equivalent to $\underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n \text{ factors of } a}$. We say that a is the **base** and n is the **exponent.**

We can also evaluate an exponential expression.

EXAMPLE 1

In-Class Example 1

Identify the base and the exponent in each expression and evaluate.
a) 3^4 b) $(-3)^4$ c) -3^4

Answer:
a) base: 3; exponent: 4; 81
b) base: -3; exponent: 4; 81
c) base: 3; exponent: 4; -81

Identify the base and the exponent in each expression and evaluate.

a) 2^4 b) $(-2)^4$ c) -2^4

Solution

a) 2^4 — 2 is the base, 4 is the exponent. Therefore, $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$.

b) $(-2)^4$ — -2 is the base, 4 is the exponent. Therefore, $(-2)^4 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 16$.

c) -2^4 — It may be very tempting to say that the base is -2. However, there are no parentheses in this expression. Therefore, 2 is the base, and 4 is the exponent. To evaluate,

$$\begin{aligned} -2^4 &= -1 \cdot 2^4 = -1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \\ &= -16 \end{aligned}$$

W Hint
In part c), you must follow the *order of operations* and evaluate the exponential expression before multiplying by -1.

BE CAREFUL

The expressions $(-a)^n$ and $-a^n$ are not always equivalent:

$$(-a)^n = \underbrace{(-a) \cdot (-a) \cdot (-a) \cdot \cdots \cdot (-a)}_{n \text{ factors of } -a}$$

$$-a^n = -1 \cdot \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n \text{ factors of } a}$$

[YOU TRY 1]

Identify the base and exponent in each expression and evaluate.

a) 5^3 b) -8^2 c) $\left(-\frac{2}{3}\right)^3$

2 Use the Product Rule for Exponents

Is there a rule to help us *multiply* exponential expressions? Let's rewrite each of the following products as a single power of the base using what we already know:

1) $2^3 \cdot 2^2 = \overbrace{2 \cdot 2 \cdot 2}^{\text{3 factors of 2}} \cdot \overbrace{2 \cdot 2}^{\text{2 factors of 2}} = 2^5$

2) $5^4 \cdot 5^3 = \overbrace{5 \cdot 5 \cdot 5 \cdot 5}^{\text{4 factors of 5}} \cdot \overbrace{5 \cdot 5 \cdot 5}^{\text{3 factors of 5}} = 5^7$

Let's summarize: $2^3 \cdot 2^2 = 2^5$, $\quad 5^4 \cdot 5^3 = 5^7$

Do you notice a pattern? *When you multiply expressions with the same base, keep the same base and add the exponents.* This is called the **product rule** for exponents.

Property Product Rule

Let a be any real number and let m and n be positive integers. Then,

$$a^m \cdot a^n = a^{m+n}$$

EXAMPLE 2

Find each product.

a) $2^2 \cdot 2^4$ b) $x^9 \cdot x^6$ c) $5c^3 \cdot 7c^9$ d) $(-k)^8 \cdot (-k) \cdot (-k)^{11}$

In-Class Example 2

Find each product.
a) $5^2 \cdot 5$ b) $y^4 \cdot y^9$
c) $-4x^5 \cdot (-10x^8)$ d) $d \cdot d^7 \cdot d^4$

Answer: a) 125 b) y^{13}
c) $40x^{13}$ d) d^{12}

W Hint

If you do not see an exponent on the base, it is assumed that the exponent is 1. Look closely at part d).

Solution

a) $2^2 \cdot 2^4 = 2^{2+4} = 2^6 = 64$ Since the bases are the same, add the exponents.

b) $x^9 \cdot x^6 = x^{9+6} = x^{15}$

c) $5c^3 \cdot 7c^9 = (5 \cdot 7)(c^3 \cdot c^9)$ Use the associative and commutative properties.
$= 35c^{12}$

d) $(-k)^8 \cdot (-k) \cdot (-k)^{11} = (-k)^{8+1+11} = (-k)^{20}$ Product rule

YOU TRY 2

Find each product.

a) $3 \cdot 3^2$ b) $y^{10} \cdot y^4$ c) $-6m^5 \cdot 9m^{11}$ d) $h^4 \cdot h^6 \cdot h^4$ e) $(-3)^2 \cdot (-3)^2$

BE CAREFUL

Can the product rule be applied to $4^3 \cdot 5^2$? **No!** The bases are not the same, so we cannot add the exponents. To evaluate $4^3 \cdot 5^2$, we would evaluate $4^3 = 64$ and $5^2 = 25$, then multiply:

$$4^3 \cdot 5^2 = 64 \cdot 25 = 1600$$

3 Use the Power Rule $(a^m)^n = a^{mn}$

What does $(2^2)^3$ mean? We can rewrite $(2^2)^3$ first as $2^2 \cdot 2^2 \cdot 2^2$.

$$2^2 \cdot 2^2 \cdot 2^2 = 2^{2+2+2} = 2^6 = 64$$

Notice that $(2^2)^3 = 2^{2+2+2}$, or $2^{2 \cdot 3}$. This leads us to the basic power rule for exponents: *When you raise a power to another power, keep the base and multiply the exponents.*

Property Basic Power Rule

Let a be any real number and let m and n be positive integers. Then,

$$(a^m)^n = a^{mn}$$

EXAMPLE 3

In-Class Example 3

Simplify using the power rule.
a) $(4^6)^3$ b) $(m^2)^5$ c) $(q^8)^7$

Answer: a) 4^{18} b) m^{10} c) q^{56}

Simplify using the power rule.

a) $(3^8)^4$ b) $(n^3)^7$ c) $((-f)^4)^3$

Solution

a) $(3^8)^4 = 3^{8\cdot 4} = 3^{32}$ b) $(n^3)^7 = n^{3\cdot 7} = n^{21}$ c) $((-f)^4)^3 = (-f)^{4\cdot 3} = (-f)^{12}$

[YOU TRY 3]

Simplify using the power rule.

a) $(5^4)^3$ b) $(j^6)^5$ c) $((-2)^3)^2$

4 Use the Power Rule $(ab)^n = a^n b^n$

Hint

This property demonstrates how you can distribute an exponent to the bases if the bases are being multiplied.

We can use another power rule to simplify an expression such as $(5c)^3$. We can rewrite and simplify $(5c)^3$ as $5c \cdot 5c \cdot 5c = 5 \cdot 5 \cdot 5 \cdot c \cdot c \cdot c = 5^3c^3 = 125c^3$. *To raise a product to a power, raise each factor to that power.*

Property Power Rule for a Product

Let a and b be real numbers and let n be a positive integer. Then,

$$(ab)^n = a^n b^n$$

BE CAREFUL

Notice that $(ab)^n = a^n b^n$ is different from $(a + b)^n$. $(a + b)^n \neq a^n + b^n$. We will study this in Chapter 6.

EXAMPLE 4

In-Class Example 4

Simplify each expression.
a) $(4x)^3$ b) $\left(\frac{1}{2}u\right)^4$
c) $(-3g)^3$ d) $(2w^4)^5$
e) $2(8mn)^2$

Answer: a) $64x^3$ b) $\frac{1}{16}u^4$
c) $-27g^3$ d) $32w^{20}$
e) $128m^2n^2$

Simplify each expression.

a) $(9y)^2$ b) $\left(\frac{1}{4}t\right)^3$ c) $(5c^2)^3$ d) $3(6ab)^2$

Solution

a) $(9y)^2 = 9^2y^2 = 81y^2$ b) $\left(\frac{1}{4}t\right)^3 = \left(\frac{1}{4}\right)^3 \cdot t^3 = \frac{1}{64}t^3$

c) $(5c^2)^3 = 5^3 \cdot (c^2)^3 = 125c^{2\cdot 3} = 125c^6$

d) $3(6ab)^2 = 3[6^2 \cdot (a)^2 \cdot (b)^2]$ The 3 is not in parentheses; therefore, it will not be squared.
$= 3(36a^2b^2)$
$= 108a^2b^2$

YOU TRY 4

Simplify.

a) $(k^4)^7$ b) $(2k^{10}m^3)^6$ c) $(-r^2s^8)^3$ d) $-4(3tu)^2$

5 Use the Power Rule $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ where $b \neq 0$

Another power rule allows us to simplify an expression like $\left(\frac{2}{x}\right)^4$. We can rewrite and simplify $\left(\frac{2}{x}\right)^4$ as $\frac{2}{x} \cdot \frac{2}{x} \cdot \frac{2}{x} \cdot \frac{2}{x} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{x \cdot x \cdot x \cdot x} = \frac{2^4}{x^4} = \frac{16}{x^4}$. *To raise a quotient to a power, raise both the numerator and denominator to that power.*

Property Power Rule for a Quotient

Let a and b be real numbers and let n be a positive integer. Then,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \text{ where } b \neq 0$$

EXAMPLE 5

In-Class Example 5

Simplify using the power rule for quotients.

a) $\left(\frac{4}{5}\right)^3$ b) $\left(\frac{x}{6}\right)^2$ c) $\left(\frac{u}{v}\right)^7$

Answer: a) $\frac{64}{125}$ b) $\frac{x^2}{36}$ c) $\frac{u^7}{v^7}$

Simplify using the power rule for quotients.

a) $\left(\frac{3}{8}\right)^2$ b) $\left(\frac{5}{x}\right)^3$ c) $\left(\frac{t}{u}\right)^9$

Solution

a) $\left(\frac{3}{8}\right)^2 = \frac{3^2}{8^2} = \frac{9}{64}$ b) $\left(\frac{5}{x}\right)^3 = \frac{5^3}{x^3} = \frac{125}{x^3}$ c) $\left(\frac{t}{u}\right)^9 = \frac{t^9}{u^9}$

YOU TRY 5

Simplify using the power rule for quotients.

a) $\left(\frac{5}{12}\right)^2$ b) $\left(\frac{2}{d}\right)^5$ c) $\left(\frac{u}{v}\right)^6$

Let's summarize the rules of exponents we have learned in this section:

Summary The Product and Power Rules of Exponents

In the rules below, a and b are any real numbers, and m and n are positive integers.

Rule		Example
Product rule	$a^m \cdot a^n = a^{m+n}$	$p^4 \cdot p^{11} = p^{4+11} = p^{15}$
Basic power rule	$(a^m)^n = a^{mn}$	$(c^8)^3 = c^{8\cdot 3} = c^{24}$
Power rule for a product	$(ab)^n = a^nb^n$	$(3z)^4 = 3^4 \cdot z^4 = 81z^4$
Power rule for a quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $(b \neq 0)$	$\left(\frac{w}{2}\right)^4 = \frac{w^4}{2^4} = \frac{w^4}{16}$

ANSWERS TO [YOU TRY] EXERCISES

1) a) base: 5; exponent: 3; $5^3 = 125$ b) base: 8; exponent: 2; $-8^2 = -64$ c) base: $-\frac{2}{3}$; exponent: 3; $\left(-\frac{2}{3}\right)^3 = -\frac{8}{27}$ 2) a) 27 b) y^{14} c) $-54m^{16}$ d) h^{14} e) 81 3) a) 5^{12} b) j^{30} c) 64
4) a) k^{28} b) $64k^{60}m^{18}$ c) $-r^6s^{24}$ d) $-36t^2u^2$ 5) a) $\frac{25}{144}$ b) $\frac{32}{d^5}$ c) $\frac{u^6}{v^6}$

E Evaluate

5.1A Exercises

Do the exercises, and check your work.

*Additional answers can be found in the Answers to Exercises appendix.

Objective 1: Evaluate Exponential Expressions

Rewrite each expression using exponents.

24 HRS 1) $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$ 9^6

2) $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ 4^7

3) $\left(\frac{1}{7}\right)\left(\frac{1}{7}\right)\left(\frac{1}{7}\right)\left(\frac{1}{7}\right)$ $\left(\frac{1}{7}\right)^4$

4) $(0.8)(0.8)(0.8)$ $(0.8)^3$

5) $(-5)(-5)(-5)(-5)(-5)(-5)(-5)$ $(-5)^7$

6) $(-c)(-c)(-c)(-c)(-c)$ $(-c)^5$

7) $(-3y)(-3y)(-3y)(-3y)(-3y)(-3y)(-3y)(-3y)$ $(-3y)^8$

8) $\left(-\frac{5}{4}t\right)\left(-\frac{5}{4}t\right)\left(-\frac{5}{4}t\right)\left(-\frac{5}{4}t\right)$ $\left(-\frac{5}{4}t\right)^4$

Identify the base and the exponent in each.

9) 6^8 base: 6; exponent: 8

10) 9^4 base: 9; exponent: 4

11) $(0.05)^7$ base: 0.05; exponent: 7

12) $(0.3)^{10}$ base: 0.3; exponent: 10

13) $(-8)^5$ base: -8; exponent: 5

14) $(-7)^6$ base: -7; exponent: 6

15) $(9x)^8$ base: $9x$; exponent: 8

16) $(13k)^3$ base: $13k$; exponent: 3

17) $(-11a)^2$ base: $-11a$; exponent: 2

18) $(-2w)^9$ base: $-2w$; exponent: 9

24 HRS 19) $5p^4$ base: p; exponent: 4

20) $-3m^5$ base: m; exponent: 5

21) $-\frac{3}{8}y^2$ base: y; exponent: 2

22) $\frac{5}{9}t^7$ base: t; exponent: 7

23) Evaluate $(3 + 4)^2$ and $3^2 + 4^2$. Are they equivalent? Why or why not?

24) Evaluate $(7 - 3)^2$ and $7^2 - 3^2$. Are they equivalent? Why or why not?

25) For any values of a and b, does $(a + b)^2 = a^2 + b^2$? Why or why not? Answers may vary.

26) Does $-2^4 = (-2)^4$? Why or why not?

27) Are $3t^4$ and $(3t)^4$ equivalent? Why or why not?
No, $3t^4 = 3 \cdot t^4$; $(3t)^4 = 3^4 \cdot t^4 = 81t^4$

28) Is there any value of a for which $(-a)^2 = -a^2$? Support your answer with an example.
Yes. If $a = 0$, then $(-0)^2 = 0$ and $-0^2 = 0$.

Evaluate.

24 HRS 29) 2^5 32

30) 9^2 81

31) $(11)^2$ 121

32) 4^3 64

33) $(-2)^4$ 16

34) $(-5)^3$ -125

35) -3^4 -81

36) -6^2 -36

37) -2^3 -8

38) -8^2 -64

39) $\left(\frac{1}{5}\right)^3$ $\frac{1}{125}$

40) $\left(\frac{3}{2}\right)^4$ $\frac{81}{16}$

For Exercises 41–44, answer *always, sometimes,* or *never*.

41) Raising a negative base to an even exponent power will *always, sometimes,* or *never* give a negative result. never

42) If the base of an exponential expression is 1, the result will *always, sometimes,* or *never* be 1. always

43) If b is any integer value except zero, then the exponential expression $(-b)^3$ will *always, sometimes,* or *never* give a negative result. sometimes

44) If a is any integer value except zero, then the exponential expression $-a^4$ will *always, sometimes,* or *never* give a positive result. never

Objective 2: Use the Product Rule for Exponents

Evaluate the expression using the product rule, where applicable.

45) $2^2 \cdot 2^3$ 32

46) $5^2 \cdot 5$ 125

47) $3^2 \cdot 3^2$ 81

48) $2^3 \cdot 2^3$ 64

49) $5^2 \cdot 2^3$ 200

50) $4^3 \cdot 3^2$ 576

24 HRS 51) $\left(\frac{1}{2}\right)^4 \cdot \left(\frac{1}{2}\right)^2$ $\frac{1}{64}$

52) $\left(\frac{4}{3}\right) \cdot \left(\frac{4}{3}\right)^2$ $\frac{64}{27}$

Simplify the expression using the product rule. Leave your answer in exponential form.

53) $8^3 \cdot 8^9$ 8^{12}

54) $6^4 \cdot 6^3$ 6^7

55) $5^2 \cdot 5^4 \cdot 5^5$ 5^{11}

56) $12^4 \cdot 12 \cdot 12^2$ 12^7

57) $(-7)^2 \cdot (-7)^3 \cdot (-7)^3$ $(-7)^8$

58) $(-3)^5 \cdot (-3) \cdot (-3)^6$ $(-3)^{12}$

59) $b^2 \cdot b^4$ b^6

60) $x^4 \cdot x^3$ x^7

61) $k \cdot k^2 \cdot k^3$ k^6

62) $n^6 \cdot n^5 \cdot n^2$ n^{13}

63) $8y^3 \cdot y^2$ $8y^5$

64) $10c^8 \cdot c^2 \cdot c$ $10c^{11}$

65) $(9m^4)(6m^{11})$ $54m^{15}$

66) $(-10p^8)(-3p)$ $30p^9$

67) $(-6r)(7r^4)$ $-42r^5$

68) $(8h^5)(-5h^2)$ $-40h^7$

69) $(-7t^6)(t^3)(-4t^7)$ $28t^{16}$

70) $(3k^2)(-4k^5)(2k^4)$ $-24k^{11}$

71) $\left(\frac{5}{3}x^2\right)(12x)(-2x^3)$ $-40x^6$

72) $\left(\frac{7}{10}y^9\right)(-2y^4)(3y^2)$ $-\frac{21}{5}y^{15}$

73) $\left(\frac{8}{21}b\right)(-6b^8)\left(-\frac{7}{2}b^6\right)$ $8b^{15}$

74) $(12c^3)\left(\frac{14}{15}c^2\right)\left(\frac{5}{7}c^6\right)$ $8c^{11}$

Mixed Exercises: Objectives 3–5

Simplify the expression using one of the power rules.

75) $(y^3)^4$ y^{12}

76) $(x^5)^8$ x^{40}

77) $(w^{11})^7$ w^{77}

78) $(a^3)^2$ a^6

79) $(3^3)^2$ 729

80) $(2^2)^2$ 16

81) $((-5)^3)^2$ $(-5)^6$

82) $((-4)^5)^3$ $(-4)^{15}$

83) $\left(\frac{1}{3}\right)^4$ $\frac{1}{81}$

84) $\left(\frac{5}{2}\right)^3$ $\frac{125}{8}$

85) $\left(\frac{6}{a}\right)^2$ $\frac{36}{a^2}$

86) $\left(\frac{v}{4}\right)^3$ $\frac{v^3}{64}$

87) $\left(\frac{m}{n}\right)^5$ $\frac{m^5}{n^5}$

88) $\left(\frac{t}{u}\right)^{12}$ $\frac{t^{12}}{u^{12}}$

89) $(10y)^4$ $10{,}000y^4$

90) $(7w)^2$ $49w^2$

91) $(-3p)^4$ $81p^4$

92) $(2m)^5$ $32m^5$

93) $(-4ab)^3$ $-64a^3b^3$

94) $(-2cd)^4$ $16c^4d^4$

95) $6(xy)^3$ $6x^3y^3$

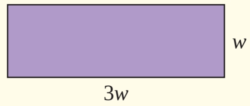
96) $-8(mn)^5$ $-8m^5n^5$

97) $-9(tu)^4$ $-9t^4u^4$

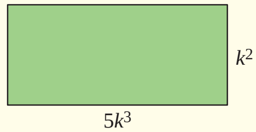
98) $2(ab)^6$ $2a^6b^6$

Mixed Exercises: Objectives 2–5

99) Find the area and perimeter of each rectangle.

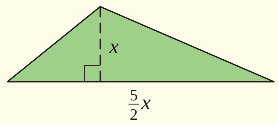
a) 

$A = 3w^2$ sq units; $P = 8w$ units

b) 

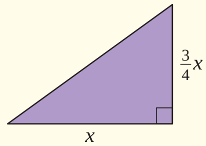
$A = 5k^5$ sq units; $P = 10k^3 + 2k^2$ units

100) Find the area.



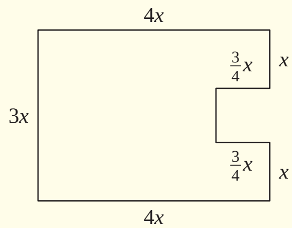
$\frac{5}{4}x^2$ sq units

101) Find the area.



$\frac{3}{8}x^2$ sq units

102) Here are the shape and dimensions of the Millers' family room. They will have wall-to-wall carpeting installed, and the carpet they have chosen costs $2.50/ft^2.



a) Write an expression for the amount of carpet they will need. (Include the correct units.) $\frac{45}{4}x^2$ ft^2

b) Write an expression for the cost of carpeting the family room. (Include the correct units.) $\frac{225}{8}x^2$ dollars

Rethink

R1) When can you "distribute" an exponent into parentheses?

R2) When can you *not* "distribute" an exponent into parentheses?

R3) When do you add the exponents?

R4) When do you multiply the exponents?

R5) Which exercises in this section do you find most challenging? Where could you go for help?

5.1B Basic Rules of Exponents: Combining the Rules

P Prepare

O Organize

What is your objective for Section 5.1B?	How can you accomplish the objective?
1 Combine the Product Rule and Power Rules of Exponents	• Be able to follow the order of operations correctly, and complete the given example on your own. • Complete You Try 1.

W Work

Read the explanations, follow the examples, take notes, and complete the You Try.

1 Combine the Product Rule and Power Rules of Exponents

When we combine the rules of exponents, we follow the order of operations.

EXAMPLE 1

In-Class Example 1

Simplify.

a) $(4f)^2(2f)^3$ b) $-3(4a^2b^3)^2$

c) $\dfrac{(12y^3)^2}{(4z^{10})^3}$

Answer: a) $128f^5$ b) $-48a^4b^6$ c) $\dfrac{9y^6}{4z^{30}}$

W Hint

Notice how the parentheses are used in these examples.

Simplify.

a) $(2c)^3(3c^8)^2$ b) $2(5k^4m^3)^3$ c) $\dfrac{(6t^5)^2}{(2u^4)^3}$

Solution

a) $(2c)^3(3c^8)^2$

Because evaluating exponents comes before multiplying in the order of operations, *evaluate the exponents first*.

$$\begin{aligned}(2c)^3(3c^8)^2 &= (2^3c^3)(3^2)(c^8)^2 && \text{Use the power rule.}\\ &= (8c^3)(9c^{16}) && \text{Use the power rule and evaluate exponents.}\\ &= 72c^{19} && \text{Product rule}\end{aligned}$$

b) $2(5k^4m^3)^3$

Which operation should be performed first, multiplying $2 \cdot 5$ or simplifying $(5k^4m^3)^3$? In the order of operations, we evaluate exponents before multiplying, so *we will begin by simplifying* $(5k^4m^3)^3$.

$$\begin{aligned}2(5k^4m^3)^3 &= 2 \cdot (5)^3(k^4)^3(m^3)^3 && \text{Order of operations and power rule}\\ &= 2 \cdot 125k^{12}m^9 && \text{Power rule}\\ &= 250k^{12}m^9 && \text{Multiply.}\end{aligned}$$

c) $\dfrac{(6t^5)^2}{(2u^4)^3}$

What comes first in the order of operations, dividing or evaluating exponents? *Evaluating exponents.*

$$\begin{aligned}\frac{(6t^5)^2}{(2u^4)^3} &= \frac{36t^{10}}{8u^{12}} && \text{Power rule}\\ &= \frac{\overset{9}{\cancel{36}}t^{10}}{\underset{2}{\cancel{8}}u^{12}} && \text{Divide out the common factor of 4.}\\ &= \frac{9t^{10}}{2u^{12}}\end{aligned}$$

BE CAREFUL

When simplifying the expression in Example 1c, $\dfrac{(6t^5)^2}{(2u^4)^3}$, it may be tempting to simplify before applying the product rule, like this:

$$\frac{(\overset{3}{\cancel{6}}t^5)^2}{(\underset{1}{\cancel{2}}u^4)^3} \neq \frac{(3t^5)^2}{(u^4)^3} = \frac{9t^{10}}{u^{12}} \quad \textbf{Wrong!}$$

You can see, however, that because we did not follow the rules for the order of operations, we did not get the correct answer.

[YOU TRY 1]

Simplify.

a) $-4(2a^9b^6)^4$ b) $(7x^{10}y)^2(-x^4y^5)^4$

c) $\dfrac{10(m^2n^3)^5}{(5p^4)^2}$ d) $\left(\dfrac{1}{6}w^7\right)^2(3w^{11})^3$

ANSWERS TO [YOU TRY] EXERCISE

1) a) $-64a^{36}b^{24}$ b) $49x^{36}y^{22}$ c) $\dfrac{2m^{10}n^{15}}{5p^8}$ d) $\dfrac{3}{4}w^{47}$

E Evaluate 5.1B Exercises

Do the exercises, and check your work.

*Additional answers can be found in the Answers to Exercises appendix.

Objective 1: Combine the Product Rule and Power Rules of Exponents

1) When evaluating expressions involving exponents, always keep in mind the order of ______. operations

2) The first step in evaluating $(9-3)^2$ is ______. subtraction

Simplify.

3) $(k^9)^2(k^3)^2$ k^{24}

4) $(d^5)^3(d^2)^4$ d^{23}

24 HRS 5) $(5z^4)^2(2z^6)^3$ $200z^{26}$

6) $(3r)^2(6r^8)^2$ $324r^{18}$

7) $6ab(-a^{10}b^2)^3$ $-6a^{31}b^7$

8) $-5pq^4(-p^4q)^4$ $-5p^{17}q^8$

9) $(9+2)^2$ 121

10) $(8-5)^3$ 27

11) $(-4t^6u^2)^3(u^4)^5$ $-64t^{18}u^{26}$

12) $(-m^2)^6(-2m^9)^4$ $16m^{48}$

13) $8(6k^7l^2)^2$ $288k^{14}l^4$

14) $5(-7c^4d)^2$ $245c^8d^2$

15) $\left(\dfrac{3}{g^5}\right)^3\left(\dfrac{1}{6}\right)^2$ $\dfrac{3}{4g^{15}}$

16) $\left(-\dfrac{2}{5}z^5\right)^3(10z)^2$ $-\dfrac{32}{5}z^{17}$

17) $\left(\dfrac{7}{8}n^2\right)^2(-4n^9)^2$ $\dfrac{49}{4}n^{22}$

18) $\left(\dfrac{2}{3}d^8\right)^4\left(\dfrac{9}{2}d^3\right)^2$ $4d^{38}$

19) $h^4(10h^3)^2(-3h^9)^2$ $900h^{28}$

20) $-v^6(-2v^5)^5(-v^4)^3$ $-32v^{43}$

21) $3w^{11}(7w^2)^2(-w^6)^5$ $-147w^{45}$

22) $5z^3(-4z)^2(2z^3)^2$ $320z^{11}$

24 HRS 23) $\dfrac{(12x^3)^2}{(10y^5)^2}$ $\dfrac{36x^6}{25y^{10}}$

24) $\dfrac{(-3a^4)^3}{(6b)^2}$ $-\dfrac{3a^{12}}{4b^2}$

25) $\dfrac{(4d^9)^2}{(-2c^5)^6}$ $\dfrac{d^{18}}{4c^{30}}$

26) $\dfrac{(-5m^7)^3}{(5n^{12})^2}$ $-\dfrac{5m^{21}}{n^{24}}$

27) $\dfrac{8(a^4b^7)^9}{(6c)^2}$ $\dfrac{2a^{36}b^{63}}{9c^2}$

28) $\dfrac{(3x^5)^3}{21(yz^2)^6}$ $\dfrac{9x^{15}}{7y^6z^{12}}$

29) $\dfrac{r^4(r^5)^7}{2t(11t^2)^2}$ $\dfrac{r^{39}}{242t^5}$

30) $\dfrac{k^5(k^2)^3}{7m^{10}(2m^3)^2}$ $\dfrac{k^{11}}{28m^{16}}$

31) $\left(\dfrac{4}{9}x^3y\right)^2\left(\dfrac{3}{2}x^6y^4\right)^3$ $\dfrac{2}{3}x^{24}y^{14}$

32) $(6s^8t^3)^2\left(-\dfrac{10}{3}st^4\right)^2$ $400s^{18}t^{14}$

33) $\left(-\dfrac{2}{5}c^9d^2\right)^3\left(\dfrac{5}{4}cd^6\right)^2$ $-\dfrac{1}{10}c^{29}d^{18}$

34) $-\dfrac{11}{12}\left(\dfrac{3}{2}m^3n^{10}\right)^2$ $-\dfrac{33}{16}m^6n^{20}$

24 HRS 35) $\left(\dfrac{5x^5y^2}{z^4}\right)^3$ $\dfrac{125x^{15}y^6}{z^{12}}$

36) $\left(-\dfrac{7a^4b}{8c^6}\right)^2$ $\dfrac{49a^8b^2}{64c^{12}}$

37) $\left(-\dfrac{3t^4u^9}{2v^7}\right)^4$ $\dfrac{81t^{16}u^{36}}{16v^{28}}$

38) $\left(\dfrac{2pr^8}{q^{11}}\right)^5$ $\dfrac{32p^5r^{40}}{q^{55}}$

39) $\left(\dfrac{12w^5}{4x^3y^6}\right)^2$ $\dfrac{9w^{10}}{x^6y^{12}}$

40) $\left(\dfrac{10b^3c^5}{15a}\right)^2$ $\dfrac{4b^6c^{10}}{9a^2}$

Determine whether the equation is *true* or *false*.

41) $(2k^2 + 2k^2) = (2k^2)^2$ false

42) $(4c^3 - 2c^2) = 2c$ false

For Exercises 43 and 44, answer *always, sometimes,* or *never*.

43) If a and b are any integer values except zero, then the exponential expression $-b^2(-a)^4$ will *always, sometimes,* or *never* give a positive result. never

44) If a and b are any integer values except zero, then the exponential expression $\left(\frac{-a}{b}\right)^3$ will *always, sometimes,* or *never* give a positive result. sometimes

45) The length of a side of a square is $5l^2$ units.

a) Write an expression for its perimeter. $20l^2$ units

b) Write an expression for its area. $25l^4$ sq units

46) The width of a rectangle is $2w$ units, and the length of the rectangle is $7w$ units.

a) Write an expression for its area. $14w^2$ sq units

b) Write an expression for its perimeter. $18w$ units

47) The length of a rectangle is x units, and the width of the rectangle is $\frac{3}{8}x$ units.

a) Write an expression for its area. $\frac{3}{8}x^2$ sq units

b) Write an expression for its perimeter. $\frac{11}{4}x$ units

48) The width of a rectangle is $4y^3$ units, and the length of the rectangle is $\frac{13}{2}y^3$ units.

a) Write an expression for its perimeter. $21y^3$ units

b) Write an expression for its area. $26y^6$ sq units

R Rethink

R1) How are the parentheses being used in these problems?

R2) Which exercises do you need to come back to and try again? How will this help you prepare for an exam?

5.2A Integer Exponents: Real-Number Bases

P Prepare **O Organize**

What are your objectives for Section 5.2A?	How can you accomplish each objective?
1 Use 0 as an Exponent	• Understand the definition of *zero as an exponent* and write it in your notes. • Complete the given example on your own. • Complete You Try 1.
2 Use Negative Integers as Exponents	• Understand the definition of *negative exponent* and write it in your notes. • Complete the given example on your own. • Complete You Try 2.

W Work

Read the explanations, follow the examples, take notes, and complete the You Trys.

So far, we have defined an exponential expression such as 2^3. The exponent of 3 indicates that $2^3 = 2 \cdot 2 \cdot 2$ (3 factors of 2) so that $2^3 = 2 \cdot 2 \cdot 2 = 8$. Is it possible to have an exponent of zero or a negative exponent? If so, what do they mean?

1 Use 0 as an Exponent

Definition

Zero as an Exponent: If $a \neq 0$, then $a^0 = 1$.

How can this be possible?

Let's evaluate $2^0 \cdot 2^3$. Using the product rule, we get:

$$2^0 \cdot 2^3 = 2^{0+3} = 2^3 = 8$$

But we know that $2^3 = 8$. Therefore, if $2^0 \cdot 2^3 = 8$, then $2^0 = 1$. This is one way to understand that $a^0 = 1$.

EXAMPLE 1

In-Class Example 1

Evaluate.

a) 3^0 b) -6^0 c) $(-4)^0$ d) $-2^0(4)$

Answer: a) 1 b) -1 c) 1 d) -4

Evaluate each expression.

a) 5^0 b) -8^0 c) $(-7)^0$ d) $-3(2^0)$

Solution

a) $5^0 = 1$

b) $-8^0 = -1 \cdot 8^0 = -1 \cdot 1 = -1$

c) $(-7)^0 = 1$

d) $-3(2^0) = -3(1) = -3$

[YOU TRY 1]

Evaluate.

a) 9^0 b) -2^0 c) $(-5)^0$ d) $3^0(-2)$

2 Use Negative Integers as Exponents

So far, we have worked with exponents that are zero or positive. What does a negative exponent mean?

Let's use the product rule to find $2^3 \cdot 2^{-3}$: $2^3 \cdot 2^{-3} = 2^{3+(-3)} = 2^0 = 1$

Remember that a number multiplied by its reciprocal is 1, and here we have that a quantity, 2^3, times another quantity, 2^{-3}, is 1. Therefore, 2^3 and 2^{-3} are reciprocals!

This leads to the definition of a negative exponent.

W Hint

Did you notice that the signs of the bases do NOT change?

Definition

Negative Exponent: If n is any integer and a and b are not equal to zero, then

$$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n} \quad \text{and} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n.$$

Therefore, to rewrite an expression of the form a^{-n} with a positive exponent, *take the reciprocal of the base and make the exponent positive.*

EXAMPLE 2

In-Class Example 2

Evaluate.

a) 3^{-3} b) $\left(\frac{4}{5}\right)^{-3}$ c) $\left(\frac{1}{3}\right)^{-2}$ d) $(-2)^{-2}$

Answer: a) $\frac{1}{27}$ b) $\frac{125}{64}$ c) 9 d) $\frac{1}{4}$

Evaluate each expression.

a) 2^{-3} b) $\left(\frac{3}{2}\right)^{-4}$ c) $\left(\frac{1}{5}\right)^{-3}$ d) $(-7)^{-2}$

Solution

a) 2^{-3}: The reciprocal of 2 is $\frac{1}{2}$, so $2^{-3} = \left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$.

b) $\left(\frac{3}{2}\right)^{-4}$: The reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$, so $\left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$.

BE CAREFUL Notice that a negative exponent does not make the answer negative!

W Hint

Before working out these examples, try to determine by inspection whether the answer is going to be positive or negative.

c) $\left(\frac{1}{5}\right)^{-3}$: The reciprocal of $\frac{1}{5}$ is 5, so $\left(\frac{1}{5}\right)^{-3} = 5^3 = 125$.

d) $(-7)^{-2}$: The reciprocal of -7 is $-\frac{1}{7}$, so

$$(-7)^{-2} = \left(-\frac{1}{7}\right)^2 = \left(-1 \cdot \frac{1}{7}\right)^2 = (-1)^2\left(\frac{1}{7}\right)^2 = 1 \cdot \frac{1^2}{7^2} = \frac{1}{49}$$

[YOU TRY 2]

Evaluate.

a) $(10)^{-2}$ b) $\left(\frac{1}{4}\right)^{-2}$ c) $\left(\frac{2}{3}\right)^{-3}$ d) -5^{-3}

ANSWERS TO [YOU TRY] EXERCISES

1) a) 1 b) -1 c) 1 d) -2 2) a) $\frac{1}{100}$ b) 16 c) $\frac{27}{8}$ d) $-\frac{1}{125}$

E Evaluate 5.2A Exercises

Do the exercises, and check your work.

*Additional answers can be found in the Answers to Exercises appendix.

Mixed Exercises: Objectives 1 and 2

24/7 1) True or False: Raising a positive base to a negative exponent will give a negative result. (Example: 2^{-4}) false

2) True or False: $8^0 = 1$. true

3) True or False: The reciprocal of 4 is $\frac{1}{4}$. true

4) True or False: $3^{-2} - 2^{-2} = 1^{-2}$. false

Evaluate.

5) 2^0 1

6) $(-4)^0$ 1

24/7 7) -5^0 -1

8) -1^0 -1

9) 0^8 0

10) $-(-9)^0$ -1

11) $(5)^0 + (-5)^0$ 2

12) $\left(\frac{4}{7}\right)^0 - \left(\frac{7}{4}\right)^0$ 0

24/7 13) 6^{-2} $\frac{1}{36}$

14) 9^{-2} $\frac{1}{81}$

15) 2^{-4} $\frac{1}{16}$

16) 11^{-2} $\frac{1}{121}$

17) 5^{-3} $\frac{1}{125}$

18) 2^{-5} $\frac{1}{32}$

19) $\left(\frac{1}{8}\right)^{-2}$ 64

20) $\left(\frac{1}{10}\right)^{-3}$ 1000

24/7 21) $\left(\frac{1}{2}\right)^{-5}$ 32

22) $\left(\frac{1}{4}\right)^{-2}$ 16

23) $\left(\frac{4}{3}\right)^{-3}$ $\frac{27}{64}$

24) $\left(\frac{2}{5}\right)^{-3}$ $\frac{125}{8}$

25) $\left(\frac{9}{7}\right)^{-2}$ $\frac{49}{81}$

26) $\left(\frac{10}{3}\right)^{-2}$ $\frac{9}{100}$

27) $\left(-\frac{1}{4}\right)^{-3}$ -64

28) $\left(-\frac{1}{12}\right)^{-2}$ 144

29) $\left(-\frac{3}{8}\right)^{-2}$ $\frac{64}{9}$

30) $\left(-\frac{5}{2}\right)^{-3}$ $-\frac{8}{125}$

31) -2^{-6} $-\frac{1}{64}$

32) -4^{-3} $-\frac{1}{64}$

33) -1^{-5} -1

34) -9^{-2} $-\frac{1}{81}$

35) $2^{-3} - 4^{-2}$ $\frac{1}{16}$

36) $5^{-2} + 2^{-2}$ $\frac{29}{100}$

37) $2^{-2} + 3^{-2}$ $\frac{13}{36}$

38) $4^{-1} - 6^{-2}$ $\frac{2}{9}$

39) $-9^{-2} + 3^{-3} + (-7)^0$ $\frac{83}{81}$

40) $6^0 - 9^{-1} + 4^0 + 3^{-2}$ 2

R Rethink

R1) When evaluating an expression with negative exponents, when do you get a negative answer?

R2) Explain the importance of fully understanding and using the order of operations in this chapter.

5.2B Integer Exponents: Variable Bases

P Prepare | **O Organize**

What are your objectives for Section 5.2B?	How can you accomplish each objective?
1 Use 0 as an Exponent	• Use the same definition of *zero as an exponent* used in the previous section. • Complete the given example on your own. • Complete You Try 1.
2 Rewrite an Exponential Expression with Positive Exponents	• Follow the explanation to understand how to rewrite an expression with positive exponents. • Learn and apply the definition of $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$. • Complete the given examples on your own. • Complete You Trys 2 and 3.

W Work

Read the explanations, follow the examples, take notes, and complete the You Trys.

1 Use 0 as an Exponent

We can apply 0 as an exponent to bases containing variables.

EXAMPLE 1

Evaluate each expression. Assume that the variable does not equal zero.

a) t^0 b) $(-k)^0$ c) $-(11p)^0$

Solution

a) $t^0 = 1$ b) $(-k)^0 = 1$ c) $-(11p)^0 = -1 \cdot (11p)^0 = -1 \cdot 1 = -1$

In-Class Example 1

Evaluate. Assume that the variable does not equal zero.
a) k^0 b) $(-m)^0$ c) $-3q^0$

Answer: a) 1 b) 1 c) -3

YOU TRY 1

Evaluate. Assume that the variable does not equal zero.

a) p^0 b) $(10x)^0$ c) $-(7s)^0$

2 Rewrite an Exponential Expression with Positive Exponents

Next, let's apply the definition of a negative exponent to bases containing variables. As in Example 1, we will assume that the variable does not equal zero since having zero in the denominator of a fraction will make the fraction undefined.

Recall that $2^{-4} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$. That is, to rewrite the expression with a positive exponent, we take the reciprocal of the base.

What is the reciprocal of x? The reciprocal is $\frac{1}{x}$.

EXAMPLE 2

Rewrite the expression with positive exponents. Assume that the variable does not equal zero.

a) x^{-6} b) $\left(\frac{2}{n}\right)^{-6}$ c) $3a^{-2}$

Solution

a) $x^{-6} = \left(\frac{1}{x}\right)^6 = \frac{1^6}{x^6} = \frac{1}{x^6}$

b) $\left(\frac{2}{n}\right)^{-6} = \left(\frac{n}{2}\right)^6$ The reciprocal of $\frac{2}{n}$ is $\frac{n}{2}$.

$= \frac{n^6}{2^6} = \frac{n^6}{64}$

c) $3a^{-2} = 3 \cdot \left(\frac{1}{a}\right)^2$ Remember, the base is a, *not* $3a$, since there are no parentheses. Therefore, the exponent of -2 applies only to a.

$= 3 \cdot \frac{1}{a^2} = \frac{3}{a^2}$

In-Class Example 2

Rewrite the expression with positive exponents. Assume that the variable does not equal zero.

a) y^{-4} b) $\left(\frac{3}{d}\right)^{-4}$ c) $7f^{-7}$

Answer: a) $\frac{1}{y^4}$ b) $\frac{d^4}{81}$ c) $\frac{7}{f^7}$

W Hint

Before working out these examples, be sure you correctly identify the base.

YOU TRY 2

Rewrite the expression with positive exponents. Assume that the variable does not equal zero.

a) m^{-4} b) $\left(\frac{1}{z}\right)^{-7}$ c) $-2y^{-3}$

How could we rewrite $\frac{x^{-2}}{y^{-2}}$ with only positive exponents? One way would be to apply the power rule for exponents: $\frac{x^{-2}}{y^{-2}} = \left(\frac{x}{y}\right)^{-2} = \left(\frac{y}{x}\right)^2 = \frac{y^2}{x^2}$

Notice that to rewrite the original expression with only positive exponents, the terms with the negative exponents "switch" their positions in the fraction. We can generalize this way:

Definition

If m and n are any integers and a and b are real numbers not equal to zero, then

$$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$$

EXAMPLE 3

In-Class Example 3

Rewrite the expression with positive exponents. Assume that the variables do not equal zero.

a) $\frac{f^{-1}}{g^{-5}}$ b) $\frac{9s^{-10}}{t^4}$ c) $c^{-2}d^{-7}$

Answer: a) $\frac{g^5}{f}$ b) $\frac{9}{s^{10}t^4}$ c) $\frac{1}{c^2d^7}$

Hint

If the exponent is already positive, do not change the position of the expression in the fraction.

Rewrite the expression with positive exponents. Assume that the variables do not equal zero.

a) $\frac{c^{-8}}{d^{-3}}$ b) $\frac{5p^{-6}}{q^7}$ c) $t^{-2}u^{-1}$ d) $\frac{2xy^{-3}}{3z^{-2}}$ e) $\left(\frac{ab}{4c}\right)^{-3}$

Solution

a) $\frac{c^{-8}}{d^{-3}} = \frac{d^3}{c^8}$ To make the exponents positive, "switch" the positions of the terms in the fraction.

b) $\frac{5p^{-6}}{q^7} = \frac{5}{p^6q^7}$ Since the exponent on q is positive, we do not change its position in the expression.

c) $t^{-2}u^{-1} = \frac{t^{-2}u^{-1}}{1}$

$= \frac{1}{t^2u^1} = \frac{1}{t^2u}$ Move $t^{-2}u^{-1}$ to the denominator to write with positive exponents.

d) $\frac{2xy^{-3}}{3z^{-2}} = \frac{2xz^2}{3y^3}$ To make the exponents positive, "switch" the positions of the factors with negative exponents in the fraction.

e) $\left(\frac{ab}{4c}\right)^{-3} = \left(\frac{4c}{ab}\right)^3$ To make the exponent positive, use the reciprocal of the base.

$= \frac{4^3c^3}{a^3b^3}$ Power rule

$= \frac{64c^3}{a^3b^3}$ Simplify.

[YOU TRY 3]

Rewrite the expression with positive exponents. Assume that the variables do not equal zero.

a) $\frac{n^{-6}}{y^{-2}}$ b) $\frac{z^{-9}}{3k^{-4}}$ c) $8x^{-5}y$ d) $\frac{8d^{-4}}{6m^2n^{-1}}$ e) $\left(\frac{3x^2}{y}\right)^{-2}$

ANSWERS TO [YOU TRY] EXERCISES

1) a) 1 b) 1 c) -1 2) a) $\frac{1}{m^4}$ b) z^7 c) $-\frac{2}{y^3}$ 3) a) $\frac{y^2}{n^6}$ b) $\frac{k^4}{3z^9}$ c) $\frac{8y}{x^5}$ d) $\frac{4n}{3m^2d^4}$ e) $\frac{y^2}{9x^4}$

*Additional answers can be found in the Answers to Exercises appendix.

Objective 1: Use 0 as an Exponent

1) Identify the base in each expression.

a) w^0 w b) $-3n^{-5}$ n

c) $(2p)^{-3}$ $2p$ d) $4c^0$ c

2) True or False: $6^0 - 4^0 = (6-4)^0$ false

Evaluate. Assume that the variables do not equal zero.

3) r^0 1 4) $(5m)^0$ 1

5) $-2k^0$ -2 6) $-z^0$ -1

7) $x^0 + (2x)^0$ 2 8) $\left(\frac{7}{8}\right)^0 - \left(\frac{3}{5}\right)^0$ 0

Objective 2: Rewrite an Exponential Expression with Positive Exponents

Rewrite each expression with only positive exponents. Assume that the variables do not equal zero.

9) d^{-3} $\frac{1}{d^3}$ 10) y^{-7} $\frac{1}{y^7}$

11) p^{-1} $\frac{1}{p}$ 12) a^{-5} $\frac{1}{a^5}$

13) $\frac{a^{-10}}{b^{-3}}$ $\frac{b^3}{a^{10}}$ 14) $\frac{h^{-2}}{k^{-1}}$ $\frac{k}{h^2}$

15) $\frac{y^{-8}}{x^{-5}}$ $\frac{x^5}{y^8}$ 16) $\frac{v^{-2}}{w^{-7}}$ $\frac{w^7}{v^2}$

17) $\frac{t^5}{8u^{-3}}$ $\frac{t^5u^3}{8}$ 18) $\frac{9x^{-4}}{y^5}$ $\frac{9}{x^4y^5}$

19) $5m^6n^{-2}$ $\frac{5m^6}{n^2}$ 20) $\frac{1}{9}a^{-4}b^3$ $\frac{b^3}{9a^4}$

21) $\frac{2}{t^{-11}u^{-5}}$ $2t^{11}u^5$ 22) $\frac{7r}{2t^{-9}u^2}$ $\frac{7rt^9}{2u^2}$

23) $\frac{8a^6b^{-1}}{5c^{-10}d}$ $\frac{8a^6c^{10}}{5bd}$ 24) $\frac{17k^{-8}h^5}{20m^{-7}n^{-2}}$ $\frac{17h^5m^7n^2}{20k^8}$

25) $\frac{2z^4}{x^{-7}y^{-6}}$ $2x^7y^6z^4$ 26) $\frac{1}{a^{-2}b^{-2}c^{-1}}$ a^2b^2c

27) $\left(\frac{a}{6}\right)^{-2}$ $\frac{36}{a^2}$ 28) $\left(\frac{3}{y}\right)^{-4}$ $\frac{y^4}{81}$

29) $\left(\frac{2n}{q}\right)^{-5}$ $\frac{q^5}{32n^5}$ 30) $\left(\frac{w}{5v}\right)^{-3}$ $\frac{125v^3}{w^3}$

31) $\left(\frac{12b}{cd}\right)^{-2}$ $\frac{c^2d^2}{144b^2}$ 32) $\left(\frac{2tu}{v}\right)^{-6}$ $\frac{v^6}{64t^6u^6}$

33) $-9k^{-2}$ $-\frac{9}{k^2}$ 34) $3g^{-5}$ $\frac{3}{g^5}$

35) $3t^{-3}$ $\frac{3}{t^3}$ 36) $8h^{-4}$ $\frac{8}{h^4}$

37) $-m^{-9}$ $-\frac{1}{m^9}$ 38) $-d^{-5}$ $-\frac{1}{d^5}$

39) $\left(\frac{1}{z}\right)^{-10}$ z^{10} 40) $\left(\frac{1}{k}\right)^{-6}$ k^6

41) $\left(\frac{1}{j}\right)^{-1}$ j 42) $\left(\frac{1}{c}\right)^{-7}$ c^7

43) $5\left(\frac{1}{n}\right)^{-2}$ $5n^2$ 44) $7\left(\frac{1}{t}\right)^{-8}$ $7t^8$

45) $c\left(\frac{1}{d}\right)^{-3}$ cd^3 46) $x^2\left(\frac{1}{y}\right)^{-2}$ x^2y^2

For Exercises 47–50, answer *always, sometimes,* or *never*.

47) If a is any integer value except zero, then the exponential expression $-a^{-2}$ will *always, sometimes,* or *never* give a negative result. always

48) If b is any integer value except zero, then the exponential expression $(-b)^{-3}$ will *always, sometimes,* or *never* give a positive result. sometimes

49) If a and b are any integer values except zero, then the exponential expression $a^0 - b^0$ will *always, sometimes,* or *never* equal zero. always

50) If a and b are any integer values except zero, then the exponential expression $(a^0b^0)^{-2}$ will *always, sometimes,* or *never* equal zero. never

Determine whether the equation is *true* or *false*.

51) $2 \div 2t^5 = t^{-5}$ true

52) $\frac{6x^5}{7} \div \frac{6x^8}{7} = \frac{7}{6}x^{-3}$ false

53) $(p^{-1} \div 3q^{-1})^{-2} = \left(\frac{q}{3p}\right)^2$ false

54) $(h^{-2} \div 4k^{-2})^2 = \left(\frac{2h}{k}\right)^{-4}$ true

R Rethink

R1) If a variable is raised to the power zero, the answer will always be what number? Assume the variable does not equal zero.

R2) Why is it useful to write expressions without negative exponents?

R3) Were there any problems you were unable to do? If so, write them down or circle them and ask your instructor for help.

5.3 The Quotient Rule

P Prepare | **O Organize**

What is your objective for Section 5.3?	How can you accomplish the objective?
1 Use the Quotient Rule for Exponents	• Learn the property for the **Quotient Rule for Exponents** and write an example in your notes. • Complete the given examples on your own. • Complete You Trys 1 and 2.

W Work

Read the explanations, follow the examples, take notes, and complete the You Trys.

1 Use the Quotient Rule for Exponents

In this section, we will discuss how to simplify the quotient of two exponential expressions with the same base. Let's begin by simplifying $\frac{8^6}{8^4}$. One way to simplify this expression is to write the numerator and denominator without exponents:

$$\frac{8^6}{8^4} = \frac{\cancel{8} \cdot \cancel{8} \cdot \cancel{8} \cdot \cancel{8} \cdot 8 \cdot 8}{\cancel{8} \cdot \cancel{8} \cdot \cancel{8} \cdot \cancel{8}} \qquad \text{Divide out common factors.}$$

$$= 8 \cdot 8 = 8^2 = 64$$

Therefore, $\frac{8^6}{8^4} = 8^2 = 64$.

Do you notice a relationship between the exponents in the original expression and the exponent we get when we simplify?

$$\frac{8^6}{8^4} = 8^{6-4} = 8^2 = 64$$

That's right. We *subtracted* the exponents.

Property Quotient Rule for Exponents

If m and n are any integers and $a \neq 0$, then

$$\frac{a^m}{a^n} = a^{m-n}$$

To divide expressions with the same base, keep the base the same and subtract the denominator's exponent from the numerator's exponent.

EXAMPLE 1

In-Class Example 1

Simplify. Assume that the variables do not equal zero.

a) $\frac{7^4}{7^2}$ b) $\frac{n^{12}}{n^4}$ c) $\frac{2}{2^{-3}}$ d) $\frac{x^3}{x^5}$ e) $\frac{4^3}{4^6}$

Answer: a) 49 b) n^8 c) 16 d) $\frac{1}{x^2}$ e) $\frac{1}{64}$

Hint

Be careful when you subtract exponents, especially when you are working with negative numbers!

Simplify. Assume that the variables do not equal zero.

a) $\frac{2^9}{2^3}$ b) $\frac{t^{10}}{t^4}$ c) $\frac{3}{3^{-2}}$ d) $\frac{n^5}{n^7}$ e) $\frac{3^2}{2^4}$

Solution

a) $\frac{2^9}{2^3} = 2^{9-3} = 2^6 = 64$ Since the bases are the same, subtract the exponents.

b) $\frac{t^{10}}{t^4} = t^{10-4} = t^6$ Because the bases are the same, subtract the exponents.

c) $\frac{3}{3^{-2}} = \frac{3^1}{3^{-2}} = 3^{1-(-2)}$ Since the bases are the same, subtract the exponents.

$= 3^3 = 27$ Be careful when subtracting the negative exponent!

d) $\frac{n^5}{n^7} = n^{5-7} = n^{-2}$ Same base; subtract the exponents.

$= \left(\frac{1}{n}\right)^2 = \frac{1}{n^2}$ Write with a positive exponent.

e) $\frac{3^2}{2^4} = \frac{9}{16}$ Because the bases are not the same, we cannot apply the quotient rule. Evaluate the numerator and denominator separately.

[YOU TRY 1]

Simplify. Assume that the variables do not equal zero.

a) $\frac{5^7}{5^4}$ b) $\frac{c^4}{c^{-1}}$ c) $\frac{k^2}{k^{10}}$ d) $\frac{2^3}{2^7}$

We can apply the quotient rule to expressions containing more than one variable. Here are more examples:

EXAMPLE 2

In-Class Example 2

Simplify. Assume that the variables do not equal zero.

a) $\frac{f^4g^{11}}{fg^3}$ b) $\frac{16p^{-6}q^4}{6p^{-2}q^3}$ c) $\frac{30a^2b^{-6}}{48a^4b^4}$

Simplify. Assume that the variables do not equal zero.

a) $\frac{x^8y^7}{x^3y^4}$ b) $\frac{12a^{-5}b^{10}}{8a^{-3}b^2}$

Solution

a) $\frac{x^8y^7}{x^3y^4} = x^{8-3}y^{7-4}$ Subtract the exponents.

$= x^5y^3$

Answer: a) f^3g^8 b) $\frac{8q}{3p^4}$ c) $\frac{5}{8a^2b^{10}}$

b) $\frac{12a^{-5}b^{10}}{8a^{-3}b^2}$ We will reduce $\frac{12}{8}$ in addition to applying the quotient rule.

$$\frac{\overset{3}{\cancel{12}}a^{-5}b^{10}}{\underset{2}{\cancel{8}}a^{-3}b^2} = \frac{3}{2}a^{-5-(-3)}b^{10-2}$$ Subtract the exponents.

$$= \frac{3}{2}a^{-5+3}b^8 = \frac{3}{2}a^{-2}b^8 = \frac{3b^8}{2a^2}$$

YOU TRY 2

Simplify. Assume that the variables do not equal zero.

a) $\frac{r^4s^{10}}{rs^3}$ b) $\frac{30m^6n^{-8}}{42m^4n^{-3}}$

ANSWERS TO YOU TRY EXERCISES

1) a) 125 b) c^5 c) $\frac{1}{k^8}$ d) $\frac{1}{16}$ 2) a) r^3s^7 b) $\frac{5m^2}{7n^5}$

Evaluate 5.3 Exercises

Do the exercises, and check your work.

*Additional answers can be found in the Answers to Exercises appendix.

Objective 1: Use the Quotient Rule for Exponents

State what is wrong with the following steps and then simplify correctly.

1) $\frac{a^5}{a^3} = a^{3-5} = a^{-2} = \frac{1}{a^2}$ You must subtract the denominator's exponent from the numerator's exponent; a^2.

2) $\frac{4^3}{2^6} = \left(\frac{4}{2}\right)^{3-6} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ You must have the same base in order to use the quotient rule; 1.

Determine whether the equation is *true* or *false*.

3) $3^{-3} \div 3^{-4} = \frac{3^3}{3^4}$ false

4) $2^{-4} \div 2^{-3} = \frac{2^3}{2^4}$ true

5) $t^7 \div t^{-5} = t^2$ false

6) $m^{10} \div m^{-3} = m^7$ false

7) $r^{-6} \div r^{-2} = r^{-4}$ true

8) $k^{-5} \div k^{-10} = k^{-5}$ false

Simplify using the quotient rule. Assume that the variables do not equal zero.

9) $\frac{d^{10}}{d^5}$ d^5

10) $\frac{z^{11}}{z^7}$ z^4

11) $\frac{m^9}{m^5}$ m^4

12) $\frac{a^6}{a}$ a^5

13) $\frac{8t^{15}}{t^8}$ $8t^7$

14) $\frac{4k^4}{k^2}$ $4k^2$

15) $\frac{6^{12}}{6^{10}}$ 36

16) $\frac{4^4}{4}$ 64

17) $\frac{3^{12}}{3^8}$ 81

18) $\frac{2^7}{2^4}$ 8

19) $\frac{2^5}{2^9}$ $\frac{1}{16}$

20) $\frac{9^5}{9^7}$ $\frac{1}{81}$

21) $\frac{5^6}{5^9}$ $\frac{1}{125}$

22) $\frac{8^4}{8^6}$ $\frac{1}{64}$

23) $\frac{10d^4}{d^2}$ $10d^2$

24) $\frac{3x^6}{x^2}$ $3x^4$

25) $\frac{20c^{11}}{30c^6}$ $\frac{2}{3}c^5$

26) $\frac{35t^7}{56t^2}$ $\frac{5}{8}t^5$

27) $\frac{y^3}{y^8}$ $\frac{1}{y^5}$

28) $\frac{m^4}{m^{10}}$ $\frac{1}{m^6}$

29) $\frac{x^{-3}}{x^6}$ $\frac{1}{x^9}$

30) $\frac{u^{-20}}{u^{-9}}$ $\frac{1}{u^{11}}$

31) $\frac{t^{-6}}{t^{-3}}$ $\frac{1}{t^3}$

32) $\frac{y^8}{y^{15}}$ $\frac{1}{y^7}$

33) $\frac{a^{-1}}{a^9}$ $\frac{1}{a^{10}}$

34) $\frac{m^{-9}}{m^{-3}}$ $\frac{1}{m^6}$

35) $\frac{t^4}{t}$ t^3

36) $\frac{c^7}{c^{-1}}$ c^8

37) $\frac{15w^2}{w^{10}}$ $\frac{15}{w^8}$

38) $\frac{-7p^3}{p^{12}}$ $-\frac{7}{p^9}$

39) $\dfrac{-6k}{k^4}$ $-\dfrac{6}{k^3}$

40) $\dfrac{21h^3}{h^7}$ $\dfrac{21}{h^4}$

47) $\dfrac{6v^{-1}w}{54v^2w^{-5}}$ $\dfrac{w^6}{9v^3}$

48) $\dfrac{3a^2b^{-11}}{18a^{-10}b^6}$ $\dfrac{a^{12}}{6b^{17}}$

41) $\dfrac{a^4b^9}{ab^2}$ a^3b^7

42) $\dfrac{p^5q^7}{p^2q^3}$ p^3q^4

49) $\dfrac{3c^5d^{-2}}{8cd^{-3}}$ $\dfrac{3}{8}c^4d$

50) $\dfrac{9x^{-5}y^2}{4x^{-2}y^6}$ $\dfrac{9}{4x^3y^4}$

43) $\dfrac{10k^{-2}l^{-6}}{15k^{-5}l^2}$ $\dfrac{2k^3}{3l^8}$

44) $\dfrac{28tu^{-2}}{14t^5u^{-9}}$ $\dfrac{2u^7}{t^4}$

51) $\dfrac{(x+y)^9}{(x+y)^2}$ $(x+y)^7$

52) $\dfrac{(a+b)^9}{(a+b)^4}$ $(a+b)^5$

45) $\dfrac{300x^7y^3}{30x^{12}y^8}$ $\dfrac{10}{x^5y^5}$

46) $\dfrac{63a^{-3}b^2}{7a^7b^8}$ $\dfrac{9}{a^{10}b^6}$

53) $\dfrac{(c+d)^{-5}}{(c+d)^{-11}}$ $(c+d)^6$

54) $\dfrac{(a+2b)^{-3}}{(a+2b)^{-4}}$ $a+2b$

R Rethink

R1) How could you change a positive exponent to a negative exponent?
R2) When do you add exponents?
R3) When do you subtract exponents?
R4) When do you multiply exponents?
R5) In what other courses have you seen exponents used?

Putting It All Together

P Prepare | **O Organize**

What is your objective?	How can you accomplish the objective?
1 Combine the Rules of Exponents	• Understand all of the **Rules of Exponents** and write the summary in your notes. • Complete the given example on your own. • Complete You Try 1.

W Work

Read the explanations, follow the examples, take notes, and complete the You Trys.

W Hint
Summarize all of the rules of exponents in your notes.

1 Combine the Rules of Exponents

Let's see how we can combine the rules of exponents to simplify expressions.

EXAMPLE 1

Simplify using the rules of exponents. Assume that all variables represent nonzero real numbers.

a) $(2t^{-6})^3(3t^2)^2$ b) $\dfrac{w^{-3} \cdot w^4}{w^6}$ c) $\left(\dfrac{12a^{-2}b^9}{30ab^{-2}}\right)^{-3}$

In-Class Example 1

Simplify using the rules of exponents. Assume that all variables represent nonzero real numbers.

a) $(4h^{-3})^2(2h)^3$ b) $\dfrac{z^9 \cdot z^{-6}}{z^4}$

c) $\left(\dfrac{21p^{-7}q^3}{28p^8q^{-4}}\right)^{-2}$

Answer: a) $\dfrac{128}{h^3}$ b) $\dfrac{1}{z}$

c) $\dfrac{16p^{30}}{9q^{14}}$

Hint

Remember to use the order of operations when simplifying expressions like these.

Solution

a) $(2t^{-6})^3(3t^2)^2$ We must follow the order of operations. Therefore, evaluate the exponents first.

$$\begin{aligned}(2t^{-6})^3 \cdot (3t^2)^2 &= 2^3t^{(-6)(3)} \cdot 3^2t^{(2)(2)} && \text{Apply the power rule.}\\ &= 8t^{-18} \cdot 9t^4 && \text{Simplify.}\\ &= 72t^{-18+4} && \text{Multiply } 8 \cdot 9 \text{ and add the exponents.}\\ &= 72t^{-14}\\ &= \frac{72}{t^{14}} && \text{Write the answer using a positive exponent.}\end{aligned}$$

b) $\dfrac{w^{-3} \cdot w^4}{w^6}$ Let's begin by simplifying the numerator:

$$\begin{aligned}\frac{w^{-3} \cdot w^4}{w^6} &= \frac{w^{-3+4}}{w^6} && \text{Add the exponents in the numerator.}\\ &= \frac{w^1}{w^6}\end{aligned}$$

Now, we can apply the quotient rule:

$$\begin{aligned}&= w^{1-6} = w^{-5} && \text{Subtract the exponents.}\\ &= \frac{1}{w^5} && \text{Write the answer using a positive exponent.}\end{aligned}$$

c) $\left(\dfrac{12a^{-2}b^9}{30ab^{-2}}\right)^{-3}$ Eliminate the negative exponent *outside* the parentheses by taking the reciprocal of the base. Notice that we have *not* eliminated the negatives on the exponents *inside* the parentheses.

$$\left(\frac{12a^{-2}b^9}{30ab^{-2}}\right)^{-3} = \left(\frac{30ab^{-2}}{12a^{-2}b^9}\right)^3$$

Next, we *could* apply the exponent of 3 to the quantity inside the parentheses. However, we could also simplify the expression inside the parentheses before cubing it.

$$\begin{aligned}\left(\frac{30ab^{-2}}{12a^{-2}b^9}\right)^3 &= \left(\frac{5}{2}a^{1-(-2)}b^{-2-9}\right)^3 && \text{Simplify } \tfrac{30}{12} \text{ and subtract the exponents.}\\ &= \left(\frac{5}{2}a^3b^{-11}\right)^3\\ &= \frac{125}{8}a^9b^{-33} && \text{Apply the power rule.}\\ &= \frac{125a^9}{8b^{33}} && \text{Write the answer using positive exponents.}\end{aligned}$$

YOU TRY 1

Simplify using the rules of exponents.

a) $\left(\dfrac{m^{12}n^3}{m^4n}\right)^4$ b) $(-p^{-5})^4(6p^7)^2$ c) $\left(\dfrac{9x^4y^{-5}}{54x^3y}\right)^{-2}$

If variables appear in the exponents, the same rules apply.

ANSWERS TO YOU TRY EXERCISES

1) a) $m^{32}n^8$ b) $\dfrac{36}{p^6}$ c) $\dfrac{36y^{12}}{x^2}$

Putting It All Together Exercises

Evaluate Do the exercises, and check your work.

*Additional answers can be found in the Answers to Exercises appendix.

Objective 1: Combine the Rules of Exponents

Use the rules of exponents to evaluate.

1) $\left(\dfrac{2}{3}\right)^4$ $\dfrac{16}{81}$

2) $(2^2)^3$ 64

3) $\dfrac{3^9}{3^5 \cdot 3^4}$ 1

4) $\dfrac{(-5)^6 \cdot (-5)^2}{(-5)^5}$ -125

5) $\left(\dfrac{10}{3}\right)^{-2}$ $\dfrac{9}{100}$

6) $\left(\dfrac{3}{7}\right)^{-2}$ $\dfrac{49}{9}$

7) $(9-6)^2$ 9

8) $(3-8)^3$ -125

9) 10^{-2} $\dfrac{1}{100}$

10) 2^{-3} $\dfrac{1}{8}$

11) $\dfrac{2^7}{2^{12}}$ $\dfrac{1}{32}$

12) $\dfrac{3^{19}}{3^{15}}$ 81

13) $\left(-\dfrac{5}{3}\right)^{-7} \cdot \left(-\dfrac{5}{3}\right)^4$

14) $\left(\dfrac{1}{8}\right)^{-2}$ 64

15) $3^{-2} - 12^{-1}$ $\dfrac{1}{36}$ $\dfrac{27}{125}$

16) $2^{-2} + 3^{-2}$ $\dfrac{13}{36}$

Simplify. Assume that all variables represent nonzero real numbers. The final answer should not contain negative exponents.

17) $-10(-3g^4)^3$ $270g^{12}$

18) $7(2d^3)^3$ $56d^9$

19) $\dfrac{33s}{s^{12}}$ $\dfrac{33}{s^{11}}$

20) $\dfrac{c^{-7}}{c^{-2}}$ $\dfrac{1}{c^5}$

21) $\left(\dfrac{2xy^4}{3x^{-9}y^{-2}}\right)^4$ $\dfrac{16}{81}x^{40}y^{24}$

22) $\left(\dfrac{a^6b^5}{10a^3}\right)^3$ $\dfrac{a^9b^{15}}{1000}$

23) $\left(\dfrac{9m^8}{n^3}\right)^{-2}$ $\dfrac{n^6}{81m^{16}}$

24) $\left(\dfrac{3s^{-6}}{r^2}\right)^{-4}$ $\dfrac{r^8s^{24}}{81}$

25) $(-b^5)^3$ $-b^{15}$

26) $(h^{11})^8$ h^{88}

27) $(-3m^5n^2)^3$ $-27m^{15}n^6$

28) $(13a^6b)^2$ $169a^{12}b^2$

29) $\left(-\dfrac{9}{4}z^5\right)\left(\dfrac{8}{3}z^{-2}\right)$ $-6z^3$

30) $(15w^3)\left(-\dfrac{3}{5}w^6\right)$ $-9w^9$

31) $\left(\dfrac{s^7}{t^3}\right)^{-6}$ $\dfrac{t^{18}}{s^{42}}$

32) $\dfrac{m^{-3}}{n^{14}}$ $\dfrac{1}{m^3n^{14}}$

33) $(-ab^3c^5)^2\left(\dfrac{a^4}{bc}\right)^3$ $a^{14}b^3c^7$

34) $\dfrac{(4v^3)^2}{(6v^8)^2}$ $\dfrac{4}{9v^{10}}$

35) $\left(\dfrac{48u^{-7}v^2}{36u^3v^{-5}}\right)^{-3}$ $\dfrac{27u^{30}}{64v^{21}}$

36) $\left(\dfrac{xy^5}{9x^{-2}y}\right)^{-2}$ $\dfrac{81}{x^6y^8}$

37) $\left(\dfrac{-3t^4u}{t^2u^{-4}}\right)^3$ $-27t^6u^{15}$

38) $\left(\dfrac{k^7m^7}{12k^{-1}m^6}\right)^2$ $\dfrac{1}{144}k^{16}m^2$

39) $(h^{-3})^6$ $\dfrac{1}{h^{18}}$

40) $(-d^4)^{-5}$ $-\dfrac{1}{d^{20}}$

41) $\left(\dfrac{h}{2}\right)^4$ $\dfrac{h^4}{16}$

42) $13f^{-2}$ $\dfrac{13}{f^2}$

43) $-7c^4(-2c^2)^3$ $56c^{10}$

44) $5p^3(4p^6)^2$ $80p^{15}$

45) $(12a^7)^{-1}(6a)^2$ $\dfrac{3}{a^5}$

46) $(9r^2s^2)^{-1}$ $\dfrac{1}{9r^2s^2}$

47) $\left(\dfrac{9}{20}r^4\right)(4r^{-3})\left(\dfrac{2}{33}r^9\right)$ $\dfrac{6}{55}r^{10}$

48) $\left(\dfrac{f^8 \cdot f^{-3}}{f^2 \cdot f^9}\right)^6$ $\dfrac{1}{f^{36}}$

49) $\dfrac{(a^2b^{-5}c)^{-3}}{(a^4b^{-3}c)^{-2}}$ $\dfrac{a^2b^9}{c}$

50) $\dfrac{(x^{-1}y^7z^4)^3}{(x^4yz^{-5})^{-3}}$ $\dfrac{x^9y^{24}}{z^3}$

51) $\dfrac{(2mn^{-2})^3(5m^2n^{-3})^{-1}}{(3m^{-3}n^3)^{-2}}$ $\dfrac{72n^3}{5m^5}$

52) $\dfrac{(4s^3t^{-1})^2(5s^2t^{-3})^{-2}}{(4s^3t^{-1})^3}$ $\dfrac{t^7}{100s^7}$

53) $\left(\dfrac{4n^{-3}m}{n^8m^2}\right)^0$ 1

54) $\left(\dfrac{7qr^4}{37r^{-19}}\right)^0$ 1

55) $\left(\dfrac{49c^4d^8}{21c^4d^5}\right)^{-2}$ $\dfrac{9}{49d^6}$

56) $\dfrac{(2x^4y)^{-2}}{(5xy^3)^2}$ $\dfrac{1}{100x^{10}y^8}$

Simplify. Assume that the variables represent nonzero integers. Write your final answer so that the exponents have positive coefficients.

57) $(p^{2c})^6$ p^{12c}

58) $(5d^{4t})^2$ $25d^{8t}$

59) $y^m \cdot y^{3m}$ y^{4m}

60) $x^{-5c} \cdot x^{9c}$ x^{4c}

61) $t^{5b} \cdot t^{-8b}$ $\dfrac{1}{t^{3b}}$

62) $a^{-4y} \cdot a^{-3y}$ $\dfrac{1}{a^{7y}}$

63) $\dfrac{25c^{2x}}{40c^{9x}}$ $\dfrac{5}{8c^{7x}}$

64) $-\dfrac{3y^{-10a}}{8y^{-2a}}$ $-\dfrac{3}{8y^{8a}}$

R Rethink

R1) Did you remember to read all the directions before starting each section of the exercises? Why is it important to read all the directions?

R2) Were you able to complete the exercises without needing much prompting or help?

R3) Were there any problems you were unable to do? If so, write them down or circle them and ask your instructor for help.

5.4 Scientific Notation

P Prepare | **O Organize**

What are your objectives for Section 5.4?	How can you accomplish each objective?
1 Multiply a Number by a Power of 10	• Learn how to multiply a number by a positive power of 10. • Learn how to multiply a number by a negative power of 10. • Complete the given examples on your own. • Complete You Trys 1 and 2.
2 Understand Scientific Notation	• Write the definition of *scientific notation* in your own words and write an example. • Complete the given example on your own. • Complete You Try 3.
3 Write a Number in Scientific Notation	• Write the procedure for **Writing a Number in Scientific Notation** in your own words. • Complete the given example on your own. • Complete You Try 4.
4 Perform Operations with Numbers in Scientific Notation	• Use the rules of exponents and the order of operations to complete the given example on your own. • Complete You Try 5.

W Work

Read the explanations, follow the examples, take notes, and complete the You Trys.

The distance from Earth to the sun is approximately 150,000,000 km. A single rhinovirus (cause of the common cold) measures 0.00002 mm across. Performing operations on very large or very small numbers like these can be difficult. This is why scientists and economists, for example, often work with such numbers in a shorthand form called *scientific notation*. Writing numbers in scientific notation together with applying rules of exponents can simplify calculations with very large and very small numbers.

1 Multiply a Number by a Power of 10

Before discussing scientific notation further, we need to understand some principles behind the notation. Let's look at multiplying numbers by positive powers of 10.

EXAMPLE 1

In-Class Example 1

Multiply.
a) 9.5×10^1
b) 0.003271×10^5

Answer: a) 95 b) 327.1

Multiply.

a) 3.4×10^1 b) $0.0857 \cdot 10^3$

Solution

a) $3.4 \times 10^1 = 3.4 \times 10 = 34$

b) $0.0857 \cdot 10^3 = 0.0857 \times 1000 = 85.7$

Notice that when we multiply each of these numbers by a positive power of 10, the result is *larger* than the original number. In fact, the exponent determines how many places to the *right* the decimal point is moved.

$$3.40 \times 10^1 = \underbrace{3.4}_{\text{1 place to the right}} \times 10^1 = 34 \qquad \underbrace{0.0857}_{\text{3 places to the right}} \cdot 10^3 = 85.7$$

YOU TRY 1

Multiply by moving the decimal point the appropriate number of places.

a) 6.2×10^2 b) $5.31 \cdot 10^5$

What happens to a number when we multiply by a *negative* power of 10?

EXAMPLE 2

In-Class Example 2

Multiply.
a) 54×10^{-3}
b) 495×10^{-7}

Answer: a) 0.054
b) 0.0000495

Multiply.

a) $41 \cdot 10^{-2}$ b) 367×10^{-4}

Solution

a) $41 \cdot 10^{-2} = 41 \cdot \dfrac{1}{100} = \dfrac{41}{100} = 0.41$

b) $367 \times 10^{-4} = 367 \times \dfrac{1}{10{,}000} = \dfrac{367}{10{,}000} = 0.0367$

When we multiply each of these numbers by a negative power of 10, the result is *smaller* than the original number. The exponent determines how many places to the *left* the decimal point is moved:

$$41 \cdot 10^{-2} = \underbrace{41.}_{\text{2 places to the left}} \cdot 10^{-2} = 0.41 \qquad 367 \times 10^{-4} = \underbrace{0367.}_{\text{4 places to the left}} \times 10^{-4} = 0.0367$$

YOU TRY 2

Multiply.

a) $83 \cdot 10^{-2}$ b) 45×10^{-3}

It is important to understand the previous concepts to understand how to use scientific notation.

2 Understand Scientific Notation

Definition

A number is in **scientific notation** if it is written in the form $a \times 10^n$, where $1 \leq |a| < 10$ and n is an integer.

Note

Multiplying $|a|$ by a *positive* power of 10 will result in a number that is *larger* than $|a|$. Multiplying $|a|$ by a *negative* power of 10 will result in a number that is *smaller* than $|a|$. The double inequality $1 \leq |a| < 10$ means that a is a number that has *one* nonzero digit to the left of the decimal point.

In other words, a number in scientific notation has one digit to the left of the decimal point and the number is multiplied by a power of 10.

Here are some examples of numbers written in scientific notation: 3.82×10^{-5}, $1.2 \cdot 10^3$, and 7×10^{-2}.

The following numbers are *not* in scientific notation:

51.94×10^4	$0.61 \cdot 10^{-3}$	$300 \cdot 10^6$
↑	↑	↑
2 digits to left of decimal point	Zero is to left of decimal point	3 digits to left of decimal point

Now let's convert a number written in scientific notation to a number without exponents.

EXAMPLE 3

In-Class Example 3

Rewrite without exponents.
a) $1.745 \cdot 10^3$ b) 1.07×10^{-2}
c) 3.904×10^4

Answer: a) 1745 b) 0.0107
c) 39,040

Hint

How does this process compare to Examples 1 and 2?

Rewrite without exponents.

a) $5.923 \cdot 10^4$ b) 7.4×10^{-3} c) 1.8875×10^3

Solution

a) $5.923 \cdot 10^4 \rightarrow 5.9230 = 59{,}230$
4 places to the right

Remember, multiplying by a positive power of 10 will make the result *larger* than 5.923.

b) $7.4 \times 10^{-3} \rightarrow 007.4 = 0.0074$
3 places to the left

Multiplying by a negative power of 10 will make the result *smaller* than 7.4.

c) $1.8875 \times 10^3 \rightarrow 1.8875 = 1887.5$
3 places to the right

YOU TRY 3 Rewrite without exponents.

a) $3.05 \cdot 10^4$ b) 8.3×10^{-5} c) 6.91853×10^3

3 Write a Number in Scientific Notation

Let's write the number 48,000 in scientific notation. First locate its decimal point: 48,000.

Next, determine where the decimal point will be when the number is in scientific notation:

48,000.

Decimal point will be here.

Therefore, $48{,}000 = 4.8 \times 10^n$, where n is an integer. Will n be positive or negative? We can see that 4.8 must be multiplied by a *positive* power of 10 to make it larger, 48,000.

48000.

Decimal point will be here. Decimal point starts here.

Now we count four places between the original and the final decimal place locations.

$\underbrace{4\,8000.}_{1234}$

We use the number of spaces, 4, as the exponent of 10.

$$48{,}000 = 4.8 \times 10^4$$

EXAMPLE 4 Write each number in scientific notation.

In-Class Example 4

Write each number in scientific notation.

a) In 2010, New York City's population was approximately 8,200,000 people.

b) The speed of a garden snail is 0.03 mph.

Answer: a) 8.2×10^6 people b) 3×10^{-2} mph

Solution

a) The distance from Earth to the sun is approximately 150,000,000 km.

150,000,000. 150,000,000. Move decimal point 8 places.

Decimal point will be here. Decimal point is here.

$$150{,}000{,}000 \text{ km} = 1.5 \times 10^8 \text{ km}$$

b) A single rhinovirus measures 0.00002 mm across.

0.00002 mm $\qquad 0.00002 \text{ mm} = 2 \times 10^{-5} \text{ mm}$

Decimal point will be here.

Procedure How to Write a Number in Scientific Notation

1) Locate the decimal point in the original number.
2) Determine where the decimal point will be when converting to scientific notation. Remember, there will be *one* nonzero digit to the left of the decimal point.
3) Count how many places you must move the decimal point to take it from its original place to its position for scientific notation.
4) If the absolute value of the resulting number is *smaller* than the absolute value of the original number, you will multiply the result by a *positive* power of 10.

$$\text{Example: } 350.9 = 3.509 \times 10^2$$

If the absolute value of the resulting number is *larger* than the absolute value of the original number, you will multiply the result by a *negative* power of 10.

$$\text{Example: } 0.0000068 = 6.8 \times 10^{-6}$$

YOU TRY 4

Write each number in scientific notation.

a) The gross domestic product of the United States in 2010 was approximately \$14,582,400,000,000. (www.worldbank.org)
b) The diameter of a human hair is approximately 0.001 in.

4 Perform Operations with Numbers in Scientific Notation

We use the rules of exponents to perform operations with numbers in scientific notation.

EXAMPLE 5

In-Class Example 5

Simplify $\dfrac{5 \times 10^4}{2 \times 10^9}$.

Answer: 0.000025 or 2.5×10^{-5}

Simplify. $\dfrac{3 \times 10^3}{4 \times 10^5}$

Solution

$$\begin{aligned} \frac{3 \times 10^3}{4 \times 10^5} &= \frac{3}{4} \times \frac{10^3}{10^5} \\ &= 0.75 \times 10^{-2} && \text{Write } \tfrac{3}{4} \text{ in decimal form.} \\ &= 7.5 \times 10^{-3} && \text{Use scientific notation.} \end{aligned}$$

or 0.0075

YOU TRY 5

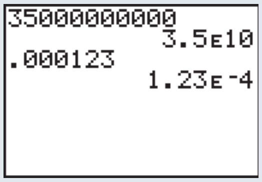
Perform the operations and simplify.

a) $(2.6 \cdot 10^2)(5 \cdot 10^4)$ b) $\dfrac{7.2 \times 10^{-9}}{6 \times 10^{-5}}$

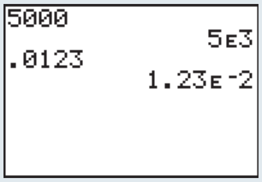
Using Technology

We can use a graphing calculator to convert a very large or very small number to scientific notation, or to convert a number in scientific notation to a number written without an exponent. Suppose we are given a very large number such as 35,000,000,000. If you enter any number with more than 10 digits on the home screen on your calculator and press $\boxed{\text{ENTER}}$, the number will automatically be displayed in scientific notation as shown on the screen below. A small number with more than two zeros to the right of the decimal point (such as .000123) will automatically be displayed in scientific notation as shown.

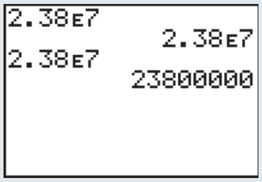
The E shown in the screen refers to a power of 10, so 3.5E10 is the number 3.5×10^{10} in scientific notation. 1.23E-4 is the number 1.23×10^{-4} in scientific notation.



If a large number has 10 or fewer digits, or if a small number has fewer than three zeros to the right of the decimal point, then the number will not automatically be displayed in scientific notation. To display the number using scientific notation, press MODE, select SCI, and press $\boxed{\text{ENTER}}$. When you return to the home screen, all numbers will be displayed in scientific notation as shown below.



A number written in scientific notation can be entered directly into your calculator. For example, the number 2.38×10^7 can be entered directly on the home screen by typing 2.38 followed by 2nd , $\boxed{7}$ $\boxed{\text{ENTER}}$ as shown here. If you wish to display this number without an exponent, change the mode back to NORMAL and enter the number on the home screen as shown.



Write each number without an exponent, using a graphing calculator.

1. 3.4×10^5
2. 9.3×10^7
3. 1.38×10^{-3}

Write each number in scientific notation, using a graphing calculator.

4. 186,000
5. 5280
6. 0.0469

ANSWERS TO [YOU TRY] EXERCISES

1) a) 620 b) 531,000 2) a) 0.83 b) 0.045 3) a) 30,500 b) 0.000083 c) 6918.53
4) a) 1.45824×10^{13} dollars b) 1.0×10^{-3} in. 5) a) 13,000,000 b) 0.00012

ANSWERS TO TECHNOLOGY EXERCISES

1) 314,000 2) 93,000,000 3) .00138 4) 1.86×10^5 5) 5.28×10^3 6) 4.69×10^{-2}

E Evaluate 5.4 Exercises

Do the exercises, and check your work.

*Additional answers can be found in the Answers to Exercises appendix.

Mixed Exercises: Objectives 1 and 2

Determine whether each number is in scientific notation.

1) 7.23×10^5 yes
2) 24.0×10^{-3} no
3) $0.16 \cdot 10^{-4}$ no
4) $-2.8 \cdot 10^4$ yes
5) -37×10^{-2} no
6) 0.9×10^{-1} no
7) $-5 \cdot 10^6$ yes
8) $7.5 \cdot 2^{-10}$ no
9) Explain, in your own words, how to determine whether a number is expressed in scientific notation. Answers may vary.
10) Explain, in your own words, how to write 4.1×10^{-3} without an exponent. Answers may vary.

Multiply.

11) $980.2 \cdot 10^4$ 9,802,000
12) $71.765 \cdot 10^2$ 7176.5
13) $0.1502 \cdot 10^8$ 15,020,000
14) $40.6 \cdot 10^{-3}$ 0.0406
15) 0.0674×10^{-1} 0.00674
16) $1{,}200{,}006 \times 10^{-7}$ 0.1200006

Objective 1: Multiply a Number by a Power of 10

Write each number without an exponent.

17) 1.92×10^6 1,920,000
18) -6.8×10^{-5} −0.000068
19) $2.03449 \cdot 10^3$ 2034.49
20) $-5.26 \cdot 10^4$ −52,600
21) $-7 \cdot 10^{-4}$ −0.0007
22) $8 \cdot 10^{-6}$ 0.000008
23) -9.5×10^{-3} −0.0095
24) 6.021967×10^5 602,196.7
25) 6×10^4 60,000
26) 3×10^6 3,000,000
27) $-9.815 \cdot 10^{-2}$ −0.09815
28) $-7.44 \cdot 10^{-4}$ −0.000744

Write the following quantities without an exponent.

29) About $2.4428 \cdot 10^7$ Americans played golf at least 2 times in a year. (Statistical Abstract of the U.S., www.census.gov) 24,428,000
30) In 2011, Facebook claimed that over 2.5×10^8 photos were uploaded each day. (www.facebook.com) 250,000,000 photos
31) The radius of one hydrogen atom is about 2.5×10^{-11} meter. 0.000000000025 meter
32) The length of a household ant is 2.54×10^{-3} meter. 0.00254 meter

Objective 3: Write a Number in Scientific Notation

Write each number in scientific notation.

33) 2110.5 2.1105×10^3
34) 38.25 3.825×10^1
35) 0.000096 9.6×10^{-5}
36) 0.00418 4.18×10^{-3}
37) −7,000,000 -7×10^6
38) 62,000 6.2×10^4
39) 3400 3.4×10^3
40) −145,000 -1.45×10^5
41) 0.0008 8×10^{-4}
42) −0.00000022 -2.2×10^{-7}
43) −0.076 -7.6×10^{-2}
44) 990 9.9×10^2
45) 6000 6×10^3
46) −500,000 -5×10^5

Write each number in scientific notation.

47) The total weight of the Golden Gate Bridge is 380,800,000 kg. (www.goldengatebridge.com) 3.808×10^8 kg
48) A typical hard drive may hold approximately 160,000,000,000 bytes of data. 1.6×10^{11} bytes
49) The diameter of an atom is about 0.00000001 cm. 1×10^{-8} cm
50) The oxygen-hydrogen bond length in a water molecule is 0.000000001 mm. 1×10^{-9} mm

Objective 4: Perform Operations with Numbers in Scientific Notation

Perform the operation as indicated. Write the final answer without an exponent.

51) $\dfrac{6 \cdot 10^9}{2 \cdot 10^5}$ 30,000
52) $(7 \cdot 10^2)(2 \cdot 10^4)$ 14,000,000
53) $(2.3 \times 10^3)(3 \times 10^2)$ 690,000
54) $\dfrac{8 \times 10^7}{4 \times 10^4}$ 2000
55) $\dfrac{8.4 \times 10^{12}}{-7 \times 10^9}$ −1200
56) $\dfrac{-4.5 \times 10^{-6}}{-1.5 \times 10^{-8}}$ 300
57) $(-1.5 \cdot 10^{-8})(4 \cdot 10^6)$ −0.06
58) $(-3 \cdot 10^{-2})(-2.6 \cdot 10^{-3})$ 0.000078
59) $\dfrac{-3 \cdot 10^5}{6 \cdot 10^8}$ −0.0005
60) $\dfrac{2 \cdot 10^1}{5 \cdot 10^4}$ 0.0004

61) $(9.75 \times 10^4) + (6.25 \times 10^4)$ 160,000

62) $(4.7 \times 10^{-3}) + (8.8 \times 10^{-3})$ 0.0135

63) $(3.19 \cdot 10^{-5}) + (9.2 \cdot 10^{-5})$ 0.0001239

64) $(2 \cdot 10^2) + (9.7 \cdot 10^2)$ 1170

For each problem, express each number in scientific notation, then solve the problem.

65) Humans shed about $1.44 \cdot 10^7$ particles of skin every day. How many particles would be shed in a year? (Assume 365 days in a year.) 5,256,000,000 particles

66) Scientists send a lunar probe to land on the moon and send back data. How long will it take for pictures to reach Earth if the distance between Earth and the moon is 360,000 km and if the speed of light is $3 \cdot 10^5$ km/sec? 1.2 sec

67) In Wisconsin in 2001, approximately 1,300,000 cows produced 2.21×10^{10} lb of milk. On average, how much milk did each cow produce? (www.nass.usda.gov) 17,000 lb/cow

68) The average snail can move 1.81×10^{-3} mi in 5 hours. What is its rate of speed in miles per hour? 0.000362 mph

24 HRS 69) A photo printer delivers approximately 1.1×10^6 droplets of ink per square inch. How many droplets of ink would a 4 in. $\times$ 6 in. photo contain? 26,400,000 droplets

70) In 2007, 3,500,000,000,000 prescription drug orders were filled in the United States. If the average price of each prescription was roughly \$65.00, how much did the United States pay for prescription drugs last year? (National Conference of State Legislatures, www.ncsl.org) \$227,500,000,000,000

71) In 2006, American households spent a total of about $7.3 \cdot 10^{11}$ dollars on food. If there were roughly 120,000,000 households in 2006, how much money did the average household spend on food? (Round to the closest dollar.) (www.census.gov) \$6083

72) Find the population density of Australia if the estimated population in 2009 was about 22,000,000 people and the country encompasses about 2,900,000 sq mi. (Australian Bureau of Statistics, www.abs.gov.au) 7.6 people/sq mi

73) According to Nielsen Media Research, over 92,000,000 people watched Super Bowl XLIII in 2009 between the Pittsburgh Steelers and the Arizona Cardinals. The California Avocado Commission estimates that about 736,000,000 ounces of avocados were eaten during that Super Bowl, mostly in the form of guacamole. On average, how many ounces of avocados did each viewer eat during the Super Bowl? 8 ounces per person

74) In 2009, the United States produced about 5.5×10^9 metric tons of carbon emissions. The U.S. population that year was about 307 million. Find the amount of carbon emissions produced per person that year. Round to the nearest tenth. (http://epa.gov, U.S. Census Bureau) 17.9 metric tons

R Rethink

R1) When would you want to write a number in scientific notation?

R2) In which college courses, other than math, do you think scientific notation is used?

Math Test Preparation Checklist

Before taking your next math test, complete the following test preparation checklist.

Test Preparation Checklist

- ❏ I checked whether it's a quiz, test, or exam.
- ❏ I began preparation at least three days before the test.
- ❏ I understand what material will be covered.
- ❏ I know how long I will have to take the test.
- ❏ I know what kinds of questions will be on the test.
- ❏ I formed or participated in a study group.
- ❏ I used my class notes and homework as study tools.
- ❏ I reviewed the relevant chapters of my textbook.
- ❏ I practiced by doing problems similar to those I will be tested on.
- ❏ I identified areas that were particularly challenging for me, and sought extra help.
- ❏ I got a good night's sleep before the test.
- ❏ I plan to make time to do some warm-up problems the day of the test.
- ❏ I have the materials I will need to complete the test: a pen or pencil, scratch paper, and a calculator (if allowed).

Chapter 5: Summary

Definition/Procedure	Example

5.1A The Product Rule and Power Rules

Exponential Expression: $a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{n \text{ factors of } a}$ a is the **base,** n is the exponent. **(p. 311)**	$5^4 = 5 \cdot 5 \cdot 5 \cdot 5$ 5 is the **base,** 4 is the exponent.
Product Rule: $a^m \cdot a^n = a^{m+n}$ **(p. 312)**	$x^8 \cdot x^2 = x^{10}$
Basic Power Rule: $(a^m)^n = a^{mn}$ **(p. 313)**	$(t^3)^5 = t^{15}$
Power Rule for a Product: $(ab)^n = a^n b^n$ **(p. 313)**	$(2c)^4 = 2^4 c^4 = 16c^4$
Power Rule for a Quotient: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, where $b \neq 0$. **(p. 314)**	$\left(\frac{w}{5}\right)^3 = \frac{w^3}{5^3} = \frac{w^3}{125}$

5.1B Combining the Rules

Remember to follow the order of operations. **(p. 318)**	Simplify $(3y^4)^2(2y^9)^3$. $= 9y^8 \cdot 8y^{27}$ Exponents come before multiplication. $= 72y^{35}$ Use the product rule and multiply coefficients.

5.2A Real-Number Bases

Zero Exponent: If $a \neq 0$, then $a^0 = 1$. **(p. 320)**	$(-9)^0 = 1$
Negative Exponent: For $a \neq 0$, $a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$. **(p. 320)**	Evaluate. $\left(\frac{5}{2}\right)^{-3} = \left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$

5.2B Variable Bases

If $a \neq 0$ and $b \neq 0$, then $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$. **(p. 323)**	Rewrite p^{-10} with a positive exponent (assume $p \neq 0$). $p^{-10} = \left(\frac{1}{p}\right)^{10} = \frac{1}{p^{10}}$
If $a \neq 0$ and $b \neq 0$, then $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$. **(p. 324)**	Rewrite each expression with positive exponents. Assume that the variables represent nonzero real numbers. a) $\frac{x^{-3}}{y^{-7}} = \frac{y^7}{x^3}$ b) $\frac{14m^{-6}}{n^{-1}} = \frac{14n}{m^6}$

5.3 The Quotient Rule

Quotient Rule: If $a \neq 0$, then $\frac{a^m}{a^n} = a^{m-n}$. **(p. 327)**	Simplify. $\frac{4^9}{4^6} = 4^{9-6} = 4^3 = 64$

Putting It All Together

Combine the Rules of Exponents (p. 329)	Simplify. $\left(\frac{a^4}{2a^7}\right)^{-5} = \left(\frac{2a^7}{a^4}\right)^5 = (2a^3)^5 = 32a^{15}$

5.4 Scientific Notation

Scientific Notation A number is in **scientific notation** if it is written in the form $a \times 10^n$, where $1 \le \lvert a \rvert < 10$ and n is an integer. That is, a is a number that has one nonzero digit to the left of the decimal point. **(p. 334)**	Write in scientific notation. a) $78{,}000 \rightarrow 78{,}000 \rightarrow 7.8 \times 10^4$ b) $0.00293 \rightarrow 0.00293 \rightarrow 2.93 \times 10^{-3}$
Converting from Scientific Notation (p. 334)	Write without exponents. a) $5 \times 10^{-4} \rightarrow 0005. \rightarrow 0.0005$ b) $1.7 \times 10^6 = 1.700000 \rightarrow 1{,}700{,}000$
Performing Operations (p. 336)	Multiply. $(4 \times 10^2)(2 \times 10^4)$ $= (4 \times 2)(10^2 \times 10^4)$ $= 8 \times 10^6$ $= 8{,}000{,}000$

Chapter 5: Review Exercises

*Additional answers can be found in the Answers to Exercises appendix.

(5.1A)

1) Write in exponential form.

a) $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$ 8^6

b) $(-7)(-7)(-7)(-7)$ $(-7)^4$

2) Identify the base and the exponent.

a) -6^5 base: 6; exponent: 5

b) $(4t)^3$ base: $4t$; exponent: 3

c) $4t^3$ base: t; exponent: 3

d) $-4t^3$ base: t; exponent: 3

3) Use the rules of exponents to simplify.

a) $2^3 \cdot 2^2$ 32

b) $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)$ $\frac{1}{27}$

c) $(7^3)^4$ 7^{12}

d) $(k^5)^6$ k^{30}

4) Use the rules of exponents to simplify.

a) $(3^2)^2$ 81

b) $8^3 \cdot 8^7$ 8^{10}

c) $(m^4)^9$ m^{36}

d) $p^9 \cdot p^7$ p^{16}

5) Simplify using the rules of exponents.

a) $(5y)^3$ $125y^3$

b) $(-7m^4)(2m^{12})$ $-14m^{16}$

c) $\left(\frac{a}{b}\right)^6$ $\frac{a^6}{b^6}$

d) $6(xy)^2$ $6x^2y^2$

e) $\left(\frac{10}{9}c^4\right)(2c)\left(\frac{15}{4}c^3\right)$ $\frac{25}{3}c^8$

6) Simplify using the rules of exponents.

a) $\left(\frac{x}{y}\right)^{10}$ $\frac{x^{10}}{y^{10}}$

b) $(-2z)^5$ $-32z^5$

c) $(6t^7)\left(-\frac{5}{8}t^5\right)\left(\frac{2}{3}t^2\right)$ $-\frac{5}{2}t^{14}$

d) $-3(ab)^4$ $-3a^4b^4$

e) $(10j^6)(4j)$ $40j^7$

(5.1B)

7) Simplify using the rules of exponents.

a) $(z^5)^2(z^3)^4$ z^{22}

b) $-2(3c^5d^8)^2$ $-18c^{10}d^{16}$

c) $(9-4)^3$ 125

d) $\frac{(10t^3)^2}{(2u^7)^3}$ $\frac{25t^6}{2u^{21}}$

8) Simplify using the rules of exponents.

a) $\left(\frac{-20d^4c}{5b^3}\right)^3$ $-\frac{64d^{12}c^3}{b^9}$

b) $(-2y^8z)^3(3yz^2)^2$ $-72y^{26}z^7$

c) $\frac{x^7 \cdot (x^2)^5}{(2y^3)^4}$ $\frac{x^{17}}{16y^{12}}$

d) $(6-8)^2$ 4

(5.2A)

9) Evaluate.

a) 8^0 1

b) -3^0 -1

c) 9^{-1} $\frac{1}{9}$

d) $3^{-2} - 2^{-2}$ $-\frac{5}{36}$

e) $\left(\frac{4}{5}\right)^{-3}$ $\frac{125}{64}$

10) Evaluate.

a) $(-12)^0$ 1

b) $5^0 + 4^0$ 2

c) -6^{-2} $-\frac{1}{36}$

d) 2^{-4} $\frac{1}{16}$

e) $\left(\frac{10}{3}\right)^{-2}$ $\frac{9}{100}$

(5.2B)

11) Rewrite the expression with positive exponents. Assume that the variables do not equal zero.

a) v^{-9} $\frac{1}{v^9}$

b) $\left(\frac{9}{c}\right)^{-2}$ $\frac{c^2}{81}$

c) $\left(\frac{1}{y}\right)^{-8}$ y^8

d) $-7k^{-9}$ $-\frac{7}{k^9}$

e) $\dfrac{19z^{-4}}{a^{-1}}$ $\dfrac{19a}{z^4}$ f) $20m^{-6}n^5$ $\dfrac{20n^5}{m^6}$

g) $\left(\dfrac{2j}{k}\right)^{-5}$ $\dfrac{k^5}{32j^5}$

12) Rewrite the expression with positive exponents. Assume that the variables do not equal zero.

a) $\left(\dfrac{1}{x}\right)^{-5}$ x^5 b) $3p^{-4}$ $\dfrac{3}{p^4}$

c) $a^{-8}b^{-3}$ $\dfrac{1}{a^8b^3}$ d) $\dfrac{12k^{-3}r^5}{16mn^{-6}}$ $\dfrac{3r^5n^6}{4k^3m}$

e) $\dfrac{c^{-1}d^{-1}}{15}$ $\dfrac{1}{15cd}$ f) $\left(-\dfrac{m}{4n}\right)^{-3}$ $-\dfrac{64n^3}{m^3}$

g) $\dfrac{10b^4}{a^{-9}}$ $10a^9b^4$

(5.3) In Exercises 13–16, assume that the variables represent nonzero real numbers. The answers should not contain negative exponents.

13) Simplify using the rules of exponents.

a) $\dfrac{3^8}{3^6}$ 9 b) $\dfrac{r^{11}}{r^3}$ r^8

c) $\dfrac{48t^{-2}}{32t^3}$ $\dfrac{3}{2t^5}$ d) $\dfrac{21xy^2}{35x^{-6}y^3}$ $\dfrac{3x^7}{5y}$

14) Simplify using the rules of exponents.

a) $\dfrac{2^9}{2^{15}}$ $\dfrac{1}{64}$ b) $\dfrac{d^4}{d^{-10}}$ d^{14}

c) $\dfrac{m^{-5}n^3}{mn^8}$ $\dfrac{1}{m^6n^5}$ d) $\dfrac{100a^8b^{-1}}{25a^7b^{-4}}$ $4ab^3$

15) Simplify by applying one or more of the rules of exponents.

a) $(-3s^4t^5)^4$ $81s^{16}t^{20}$ b) $\dfrac{(2a^6)^5}{(4a^7)^2}$ $2a^{16}$

c) $\left(\dfrac{z^4}{y^3}\right)^{-6}$ $\dfrac{y^{18}}{z^{24}}$ d) $(-x^3y)^5(6x^{-2}y^3)^2$ $-36x^{11}y^{11}$

e) $\left(\dfrac{cd^{-4}}{c^8d^{-9}}\right)^5$ $\dfrac{d^{25}}{c^{35}}$ f) $\left(\dfrac{14m^5n^5}{7m^4n}\right)^3$ $8m^3n^{12}$

g) $\left(\dfrac{3k^{-1}t}{5k^{-7}t^4}\right)^{-3}$ $\dfrac{125t^9}{27k^{18}}$ h) $\left(\dfrac{40}{21}x^{10}\right)(3x^{-12})\left(\dfrac{49}{20}x^2\right)$ 14

16) Simplify by applying one or more of the rules of exponents.

a) $\left(\dfrac{4}{3}\right)^8\left(\dfrac{4}{3}\right)^{-2}\left(\dfrac{4}{3}\right)^{-3}$ $\dfrac{64}{27}$ b) $\left(\dfrac{k^{10}}{k^4}\right)^3$ k^{18}

c) $\left(\dfrac{x^{-4}y^{11}}{xy^2}\right)^{-2}$ $\dfrac{x^{10}}{y^{18}}$ d) $(-9z^5)^{-2}$ $\dfrac{1}{81z^{10}}$

e) $\left(\dfrac{g^2 \cdot g^{-1}}{g^{-7}}\right)^{-4}$ $\dfrac{1}{g^{32}}$ f) $(12p^{-3})\left(\dfrac{10}{3}p^5\right)\left(\dfrac{1}{4}p^2\right)^2$ $\dfrac{5}{2}p^6$

g) $\left(\dfrac{30u^2v^{-3}}{40u^7v^{-7}}\right)^{-2}$ $\dfrac{16u^{10}}{9v^8}$ h) $-5(3h^4k^9)^2$ $-45h^8k^{18}$

Simplify. Assume that the variables represent nonzero integers. Write your final answer so that the exponents have positive coefficients.

17) $y^{3k} \cdot y^{7k}$ y^{10k} 18) $\dfrac{z^{12c}}{z^{5c}}$ z^{7c}

(5.4) Write each number without an exponent.

19) 9.38×10^5 938,000 20) -4.185×10^2 -418.5

21) $9 \cdot 10^3$ 9000 22) $6.7 \cdot 10^{-4}$ 0.00067

23) 1.05×10^{-6} 0.00000105 24) 2×10^4 20,000

Write each number in scientific notation.

25) 0.0000575 5.75×10^{-5} 26) 36,940 3.694×10^4

27) 32,000,000 3.2×10^7 28) 0.0000004 4×10^{-7}

29) 0.0009315 9.315×10^{-4} 30) 66 6.6×10^1

Write the number without exponents.

31) Before 2010, golfer Tiger Woods earned over 7×10^7 dollars per year in product endorsements. (www.forbes.com) 70,000,000

Perform the operation as indicated. Write the final answer without an exponent.

32) $\dfrac{8 \cdot 10^6}{2 \cdot 10^{13}}$ 0.0000004 33) $\dfrac{-1 \cdot 10^9}{5 \cdot 10^{12}}$ -0.0002

34) $(9 \times 10^{-8})(4 \times 10^7)$ 3.6 35) $(5 \cdot 10^3)(3.8 \cdot 10^{-8})$ 0.00019

36) $\dfrac{-3 \times 10^{10}}{-4 \times 10^6}$ 7500 37) $(-4.2 \times 10^2)(3.1 \times 10^3)$ $-1{,}302{,}000$

For each problem, write each of the numbers in scientific notation, then solve the problem. Write the answer without exponents.

38) Eight porcupines have a total of about $2.4 \cdot 10^5$ quills on their bodies. How many quills would one porcupine have? 30,000 quills

39) In 2010, North Dakota had approximately 4.0×10^7 acres of farmland and about 32,000 farms. What was the average size of a Nebraska farm in 2010? (www.census.gov) 1250 acres

40) One molecule of water has a mass of 2.99×10^{-23} g. Find the mass of 100,000,000 molecules. 0.00000000000000299 g

41) At the end of 2008, the number of SMS text messages sent in one month in the United States was 110.4 billion. If 270.3 million people used SMS text messaging, about how many did each person send that month? (Round to the nearest whole number.) (www.ctia.org/advocacy/research/index.cfm/AID/10323) 408 texts per person

42) When the polls closed on the west coast on November 4, 2008, and Barack Obama was declared the new president, there were about 143,000 visits per second to news websites. If the visits continued at that rate for 3 minutes, how many visits did the news websites receive during that time? (www.xconomy.com) 25,740,000

Chapter 5: Test

*Additional answers can be found in the Answers to Exercises appendix.

Write in exponential form.

1) $(-3)(-3)(-3)$ $(-3)^3$

2) $x \cdot x \cdot x \cdot x \cdot x$ x^5

Use the rules of exponents to simplify.

3) $5^2 \cdot 5$ 125

4) $\left(\frac{1}{x}\right)^5 \cdot \left(\frac{1}{x}\right)^2$ $\frac{1}{x^7}$

5) $(8^3)^{12}$ 8^{36}

6) $p^7 \cdot p^{-2}$ p^5

Evaluate.

7) 3^4 81

8) 8^0 1

9) 2^{-5} $\frac{1}{32}$

10) $4^{-2} + 2^{-3}$ $\frac{3}{16}$

11) $\left(-\frac{3}{4}\right)^3$ $-\frac{27}{64}$

12) $\left(\frac{10}{7}\right)^{-2}$ $\frac{49}{100}$

Simplify using the rules of exponents. Assume that all variables represent nonzero real numbers. The final answer should not contain negative exponents.

13) $(5n^6)^3$ $125n^{18}$

14) $(-3p^4)(10p^8)$ $-30p^{12}$

15) $\frac{m^{10}}{m^4}$ m^6

16) $\frac{a^9b}{a^5b^7}$ $\frac{a^4}{b^6}$

17) $\left(\frac{-12t^{-6}u^8}{4t^5u^{-1}}\right)^{-3}$ $-\frac{t^{33}}{27u^{27}}$

18) $(2y^{-4})^6\left(\frac{1}{2}y^5\right)^3$ $\frac{8}{y^9}$

19) $\left(\frac{(9x^2y^{-2})^3}{4xy}\right)^0$ 1

20) $\frac{(2m+n)^3}{(2m+n)^2}$ $2m+n$

21) $\frac{12a^4b^{-3}}{20c^{-2}d^3}$ $\frac{3a^4c^2}{5b^3d^3}$

22) $\left(\frac{y^{-7} \cdot y^3}{y^5}\right)^{-2}$ y^{18}

23) Simplify $t^{10k} \cdot t^{3k}$. Assume that the variables represent nonzero integers. t^{13k}

24) Rewrite $7.283 \cdot 10^5$ without exponents. 728,300

25) Write 0.000165 in scientific notation. 1.65×10^{-4}

26) Divide. Write the answer without exponents. $\frac{-7.5 \times 10^{12}}{1.5 \times 10^8}$ $-50{,}000$

27) Write the number without an exponent: In 2010, the population of Texas was about $2.51 \cdot 10^7$. (www.census.gov) 25,100,000

28) An electron is a subatomic particle with a mass of 9.1×10^{-28} g. What is the mass of 2,000,000,000 electrons? Write the answer without exponents. 0.00000000000000000182 g

Chapter 4: Cumulative Review for Chapters 1–5

*Additional answers can be found in the Answers to Exercises appendix.

Perform the indicated operations. Write the answer in lowest terms.

1) $\frac{2}{15} + \frac{1}{10} + \frac{7}{20}$ $\frac{7}{12}$

2) $\frac{4}{15} \div \frac{20}{21}$ $\frac{7}{25}$

3) $-26 + 5 - 7$ -28

4) $(5+1)^2 - 2[17 + 5(10 - 14)]$ 42

5) Glen Crest High School is building a new football field. The dimensions of a regulation-size field are $53\frac{1}{3}$ yd by 120 yd (including the 10 yd of end zone on each end). The sod for the field will cost \$1.80/yd^2.

a) Find the perimeter of the field. $346\frac{2}{3}$ yd

b) How much will it cost to sod the field? \$11,520

6) Evaluate $2p^2 - 11q$ when $p = 3$ and $q = -4$. 62

7) Given this set of numbers $\left\{3, -4, -2.1\overline{3}, \sqrt{11}, 2\frac{2}{3}\right\}$, list the

a) integers $-4, 3$

b) irrational numbers $\sqrt{11}$

c) natural numbers 3

d) rational numbers $3, -4, -2.1\overline{3}, 2\frac{2}{3}$

e) whole numbers 3

8) Combine like terms and simplify: $5(t^2 + 7t - 3) - 2(4t^2 - t + 5)$ $-3t^2 + 37t - 25$

9) Let x represent the unknown quantity, and write a mathematical expression for "thirteen less than half of a number." $\frac{1}{2}x - 13$

Solve.

10) $8t - 17 = 10t + 6$ $\left\{-\frac{23}{2}\right\}$

11) $\frac{x+3}{10} = \frac{2x-1}{4}$ $\left\{\frac{11}{8}\right\}$

Solve. Write the answer in interval notation.

12) $k - 2(3k - 7) \le 3(k + 4) - 6$ $[1, \infty)$

13) Write the equation of the line containing the points $(-8, -2)$ and $(6, 5)$. Write the answer in slope-intercept form. $y = \frac{1}{2}x + 2$

Determine whether the two lines are parallel, perpendicular, or neither.

14) $5y - 3x = 1$
$3x - 5y = -8$ parallel

Solve each system of equations.

15) $9x - 2y = 8$
$y = 2x + 1$ $(2, 5)$

16) $y = -6x + 5$
$12x + 2y = 10$ $\{(x, y) \mid y = -6x + 5\}$

Simplify using the rules of exponents. The answer should not contain negative exponents. Assume that the variables represent nonzero real numbers.

17) $4^3 \cdot 4^7$ 4^{10}

18) $\left(\frac{x}{y}\right)^{-3}$ $\frac{y^3}{x^3}$

19) $\left(\frac{32x^3}{8x^{-2}}\right)^{-1}$ $\frac{1}{4x^5}$

20) $-(3rt^{-3})^4$ $-\frac{81r^4}{t^{12}}$

21) $(4z^3)(-7z^5)$ $-28z^8$

22) $\frac{n^2}{n^9}$ $\frac{1}{n^7}$

23) $(-2a^{-6}b)^5$ $-\frac{32b^5}{a^{30}}$

24) Write 0.000729 in scientific notation. 7.29×10^{-4}

25) Perform the indicated operation. Write the final answer without an exponent. $(6.2 \times 10^5)(9.4 \times 10^{-2})$ 58,280